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VECTORS, MATRICES and COMPLEX NUMBERS

by

Jean-Paul GINESTIER and John EGSGARD

with
International Baccalaureate
questions

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A Search for Meaning

VECTORS, MATRICES and COMPLEX NUMBERS
with International Baccalaureate questions

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ABOUT THE AUTHORS

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Originally French, Jean-Paul Ginestier was brought up in England and took his first degree there. He then taught in Cameroon, then the UK, before settling with his wife and two children in Canada, where he gained teaching and administrative experience in both the private and public systems, in English and in French. He also took his B.Ed, then his M.Sc. at the University of Toronto. In 1984-85, he was on exchange at the International School of Geneva, Switzerland, where John Egsgard had taught in the same capacity two years earlier.

Jean-Paul Ginestier also has a long experience, both as a teacher and an examiner, with the International Baccalaureate. From 1988 to 1995, he was Deputy Chief Examiner for IB Higher Level Mathematics. He has also made presentations at a many mathematics conferences in a number of countries.

Since 1995 he has been Head of Mathematics at three United World Colleges - the Nordic College in Norway, then the UWC of the Adriatic in Italy from 2004-2009, then UWC Costa Rica in 2012-2014. He is now retired.

John Christian Egsgard, M.A.

John Egsgard retired from teaching in 1998 after 47 years in secondary school classrooms. Besides being an author of 15 secondary mathematics texts and editor of the Ontario Mathematics Gazette, he has been active in mathematics associations around the world. He has been president of both the National Council of Teachers of Mathematics of the USA, and the Ontario Association for Mathematics Education in Canada. John has been on many committees preparing for the first 9 International Congresses of Mathematics Education throughout the world. He has delivered papers or major speeches at most of the ICMEs.

John's experience with the IB comes from his year of teaching at the International School of Geneva, and in the marking of exams and preparing questions for them.

John passed away on July 10, 2016.

INTRODUCTION

The International Baccalaureate is a high quality programme of study that tries to bring together the best features inspired from various educational systems around the world; the IB Diploma is recognized for university entrance in most countries. In this book, past IB examination questions can be found in each of the Review Exercises and the Problem Supplement. The questions have been selected from papers for Standard Level Mathematical Studies (SMS), Standard Level Mathematics (S) and Higher Level Mathematics (H).

The original edition of this book, published by Gage (Canada Publishing Corporation) in 1989 was designed primarily to cover the Ontario Academic Credit called Algebra and Geometry. It so happens that this course was almost entirely a subset of IB Higher Level Mathematics at that time. The coverage of Vectors and Complex Numbers is thorough, and clearly explained for students in their last years of secondary education.

Teachers will find that the most relevant chapters for the IB HL Mathematics course are 1, 3, 5 and 6, although some of the notions in chapter 2 are necessary. All Matrix and Transformation work is found in chapter 7, and all Complex Numbers in chapter 10. Chapter 9 on Induction also covers a section of the IB HL Mathematics syllabus.

One characteristic of this book is that it tries to pay tribute, both by taking a historical approach where appropriate (as in the case of Complex Numbers), and by mentioning their names, to the many mathematicians who have led the evolution of these ideas for the last 500 years or so.

The book was reprinted with modifications in 1994. Although it has become the norm to publish entirely new textbooks as often as possible, then to discard them as quickly as possible, the authors believe that the best textbooks are those that are continuously revised, corrected, and updated.

Jean-Paul Ginestier
Toronto, Canada
August 2012

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Text Organization

- The text is divided into 10 chapters.
- *In Search of* sections within each chapter encourage students to investigate and explore challenging topics individually, or in small groups.
- *Making Connections* pages relate mathematics to other fields and disciplines, and encourage a greater understanding of the nature and purpose of mathematics.
- The *problem supplement* at the end of the text provides an opportunity for students to synthesize the skills and ideas acquired throughout the course.
- The *answer key* provides answers for all exercises, reviews, and inventories, as well as for the problem supplement.
- The *glossary* provides definitions of relevant mathematical terms.
- The *index* lists topics and main concepts for easy reference.

Chapter Organization

- Each chapter begins with a discussion of a problem which can be solved using the mathematics developed throughout the chapter.
- Teaching material is clearly separated from exercise material.
- Worked examples enhance the understanding of each topic.
- Colour is used to highlight generalizations, rules, and formulas.
- Mathematical terms appear in **boldface** type when they are first introduced. All relevant terms are defined in the glossary.
- *Italic* type is used for emphasis.
- Exercise material is carefully sequenced from questions that utilize and apply knowledge to those that develop critical thinking skills.
- The main concepts covered in each chapter are listed concisely in the *Chapter Summary*.
- The *Chapter Inventory* provides students with an opportunity to test their skills and understanding of the mathematical concepts in the chapter.
- The *Chapter Review* provides additional opportunities for students to apply their problem-solving skills.
- IB questions (numbered in colour) are to be found at the end of chapters, and in the problem supplement.

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