VECTORS, MATRICES and COMPLEX NUMBERS

with International Baccalaureate questions

John EGSGARD
and
Jean-Paul GINESTIER

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PROBLEM SUPPLEMENT

1. In quadrilateral $ABCD$, E and F are the midpoints of the diagonals AC and BD respectively. P is the midpoint of EF. If O is any point, prove that
$4\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD}$.

2. $ABCD$ is a rectangle. The midpoints of sides AB, BC, CD, and DA are M, N, P, and Q respectively. Prove that $MNPQ$ is a rhombus.

3. Given any triangle ABC, and any point O, a point G is positioned so that
$\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} = \vec{0}$.
Use vector subtraction, with origin O, to prove that
$\overrightarrow{OG} = \frac{1}{3}(\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC})$
(The point G is known as the **centroid** or **centre of mass** of the triangle ABC.)

4. Determine whether or not the three vectors in each of the following are linearly dependent. In each case state the geometric significance of the result.

a) $\overrightarrow{(2,0,6)}$, $\overrightarrow{(1,1,-3)}$, and $\overrightarrow{(2,1,-1)}$

b) $\overrightarrow{(3,2,1)}$, $\overrightarrow{(4,11,6)}$, and $\overrightarrow{(14,1,0)}$

c) $\overrightarrow{(4,9,1)}$, $\overrightarrow{(-3,1,1)}$, and $\overrightarrow{(6,29,3)}$

5. Given the vectors $\vec{a} = \overrightarrow{(5,-6)}$ and $\vec{b} = \overrightarrow{(4,2)}$.

a) Prove the two vectors form a basis for $\mathbb{V}_2$.

b) Express the vector $\overrightarrow{(3,-7)}$ as a linear combination of $\vec{a}$ and $\vec{b}$.

6. Express the vector $\vec{a} = \overrightarrow{(7,8,16)}$ as a linear combination of the vectors $\vec{b} = \overrightarrow{(1,-2,3)}$, $\vec{c} = \overrightarrow{(0,2,5)}$, and $\vec{d} = \overrightarrow{(2,2,1)}$.

7. Find the value for m if the vectors $\vec{a} = \overrightarrow{(2,7,-4)}$, $\vec{b} = \overrightarrow{(4,m,3)}$, and $\vec{c} = \overrightarrow{(0,1,-2)}$ are coplanar.

8. a) A point P divides the line segment AB internally in the ratio $3:5$. Express $\overrightarrow{OP}$ in terms of $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

b) If point Q divides segment PB of part a) externally in the ratio $7:4$, then express $\overrightarrow{OQ}$ in terms of $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

9. a) Four points M, K, T, and R are given such that $\overrightarrow{TR} = -\frac{3}{2}\overrightarrow{TK} + \frac{5}{2}\overrightarrow{TM}$. Draw conclusions about the points M, K, T, and R.

b) If T has coordinates $(0,0,0)$, while K and M have respectively coordinates $(1,-3,4)$ and $(5,0,2)$, find the coordinates of point R.

10. Points A, B, C, and D are points in 3-space with position vectors $\vec{a}$, $\vec{b}$, $\vec{c}$, and $\vec{d}$. $\vec{a}$, $\vec{b}$ and $\vec{c}$ are linearly independent and $\vec{d} = -7\vec{a} + 3\vec{b} + 5\vec{c}$. Prove that points A, B, C, and D are coplanar.

11. $MTRS$ is a parallelogram. Point A divides TR internally in the ratio $8:5$. Segments MR and SA intersect at point B. Use vector methods to find the ratio into which point B divides segment MR.

12. Given the vectors $\vec{a} = \overrightarrow{(-1,-4,7)}$, $\vec{u} = \overrightarrow{(2,3,-1)}$, and $\vec{v} = \overrightarrow{(4,1,11)}$.

a) Prove the vectors are linearly dependent.

b) Find a vector coplanar with $\vec{u}$ and $\vec{v}$, that is perpendicular to vector $\vec{a}$.

13. Given the vectors $\vec{u} = \overrightarrow{(1,1,1)}$, $\vec{v} = \overrightarrow{(1,-2,1)}$, and $\vec{w} = \overrightarrow{(2,0,-1)}$.

a) Prove that $\vec{u}$, $\vec{v}$, and $\vec{w}$ form a basis for $\mathbb{V}_3$.

b) Find the components of $\vec{a} = \overrightarrow{(5,3,2)}$ with respect to the basis in part a).

c) If $m\vec{u} + p\vec{v} + r\vec{w} = \overrightarrow{(x,y,z)}$, then show that $m + p + r = 1$ if and only if $2x + z = 3$.

14. Points P, D, and R are collinear and O is any point such that $\overrightarrow{OD} = 3m\overrightarrow{OP} + 4k\overrightarrow{OR}$, and $2m - 3k = 16$. Find the values of k and m.

15. Prove that the components of a vector $\vec{d}$ with respect to the $\mathbb{V}_3$ basis $\{\vec{a},\vec{b},\vec{c}\}$ are unique.

16. Express $\vec{e} = \overrightarrow{(11,6,1)}$ as a linear combination of $\vec{a} = \overrightarrow{(3,1,0)}$, $\vec{b} = \overrightarrow{(-2,0,4)}$, $\vec{c} = \overrightarrow{(0,1,2)}$, and $\vec{d} = \overrightarrow{(1,1,1)}$.

17. Prove that $\vec{a}$, $\vec{b}$, and $\vec{c} = k\vec{b}$, $k \in \mathbb{R}$ are linearly dependent in $\mathbb{V}_3$.

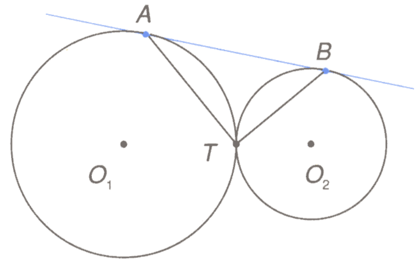
18. G is the point of intersection of the medians of a triangle ABC. The point G divides the medians AD, BE, and CF internally in the ratio $2:1$. Prove that $\overrightarrow{AG} + \overrightarrow{BG} + \overrightarrow{CG} = \vec{0}$

19. O, P, D, R, and S are five points in 3-space such that $\overrightarrow{OP} = 2\overrightarrow{OD} + 2\overrightarrow{OR} - 3\overrightarrow{OS}$. Prove that the four points P, D, R, and S are coplanar.

20. O, A, B, and C are four points in 3-space such that $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, $\overrightarrow{OC} = \vec{c}$ where $\vec{a}$, $\vec{b}$, and $\vec{c}$ are linearly independent. If D lies in the plane determined by points A, B, and C, then express $\overrightarrow{OD}$ in terms of $\vec{a}$, $\vec{b}$, and $\vec{c}$.
(You will need to use two scalars.)

21. Triangle ABC is right-angled at B, and M is the midpoint of AC. If $\overrightarrow{BA} = \vec{a}$ and $\overrightarrow{BC} = \vec{c}$, express $\overrightarrow{BM}$ and $\overrightarrow{AM}$ in terms of $\vec{a}$ and $\vec{c}$, and hence show that $|\overrightarrow{AM}| = |\overrightarrow{BM}|$.

22. Two circles, of centres O_1 and O_2, touch externally at T. A common tangent to the two circles touches them at A and B respectively. Use the dot product to prove that ATB is a right angle. (You may assume that O_1TO_2 is a straight line and that the angles O_1AB and O_2BA are right angles.)



23. $OABC$ is a tetrahedron with OA perpendicular to BC, and OC perpendicular to AB. If $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, and $\overrightarrow{OC} = \vec{c}$, express $\overrightarrow{AB}$ and $\overrightarrow{BC}$ in terms of $\vec{a}$, $\vec{b}$, and $\vec{c}$, and hence prove that $\overrightarrow{OB}$ is perpendicular to $\overrightarrow{AC}$.

24. A right square pyramid $ABCDT$ whose base has a side $2s$ and whose height is h is positioned in a three-space coordinate system as follows. $A(s,s,0)$, $B(-s,s,0)$, $C(-s,-s,0)$, $D(s,-s,0)$ and $T(0,0,h)$.

a) If θ is the angle between a slant edge and the base, show that

$$\cos\theta = \frac{h}{\sqrt{2s^2 + h^2}}$$

b) If ϕ is the angle between two triangular faces, show that

$$\cos\phi = -\frac{s^2}{s^2 + h^2}$$

25. If $\vec{w} = (\vec{v} \cdot \vec{e})\vec{e}$ and $\vec{z} + \vec{w} = \vec{v}$, prove that $\vec{z} \cdot \vec{e} = 0$. Draw a figure that shows how the vectors $\vec{v}$, $\vec{e}$, $\vec{w}$, and $\vec{z}$ are related.

26. Resolve the vector $\vec{v} = \overrightarrow{(-2,3)}$ onto the vectors $\vec{a} = \overrightarrow{(1,1)}$ and $\vec{b} = \overrightarrow{(-1,1)}$.

27. In 3.2 Exercises you proved that for any vectors $\vec{u}$ and $\vec{v}$, $|\vec{u} \cdot \vec{v}| \leqslant |\vec{u}||\vec{v}|$. Given that $\vec{u} = \overrightarrow{(u_1,u_2,u_3)}$, and $\vec{v} = \overrightarrow{(v_1,v_2,v_3)}$, write out this inequality using components. (This is known as the **Cauchy-Schwarz inequality**.)

28. The vector $\vec{n} = \overrightarrow{(2,4,-3)}$ is normal (that is, perpendicular) to the plane Π. $A(1,5,5)$ is a given point of Π, and $P(x,y,z)$ is any point of Π. Let the position vectors of A and P be $\vec{a}$ and $\vec{r}$ respectively.

a) Explain why $\overrightarrow{AP} \cdot \vec{n}$ must be zero.

b) Substitute the given values into the equation $\overrightarrow{AP} \cdot \vec{n} = 0$ to show that an equation representing the plane Π is $2x + 4y - 3z = 7$.

29. A motor boat is travelling upstream at full power. As the boat passes under a low bridge, the woman at the helm loses her hat. However, she does not notice this until 6 minutes later. At this time, she turns her boat around and goes downstream at full power. She retrieves her hat 1 km downstream from the bridge where it got knocked off. Calculate the speed of the current.

30. The diagonals of a parallelogram can be represented by the vectors $\overrightarrow{(6,6,0)}$ and $\overrightarrow{(1,-1,2)}$.

a) Prove that the parallelogram is a rhombus.

b) Calculate the length of its sides and the values of its angles.

31. Given any four points A, B, C, D in 3-space, prove that $\overrightarrow{AB} \cdot \overrightarrow{CD} + \overrightarrow{AC} \cdot \overrightarrow{BD} + \overrightarrow{AD} \cdot \overrightarrow{BC} = 0$. Deduce from this that the three altitudes of a triangle are concurrent. (That is, the three altitudes intersect at the same point.)

32. OAB is an isosceles triangle with $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, and $|\vec{a}| = |\vec{b}|$. M is the midpoint of OA, and N is the midpoint of OB. Express $\overrightarrow{AN}$ and $\overrightarrow{BM}$ in terms of $\vec{a}$ and $\vec{b}$, and use the dot product to show that $|\overrightarrow{AN}| = |\overrightarrow{BM}|$.

33. Two forces $\vec{E}$ and $\vec{F}$ act upon a particle in such a way that the resultant force $\vec{R}$ has a magnitude equal to that of $\vec{E}$, and makes an angle of 90° with $\vec{E}$. Given that $|\vec{E}| = |\vec{R}| = 50$ N, calculate the magnitude and direction of the second force relative to $\vec{E}$.

34. A river flows due east with a speed of 2.5 km/h. A woman rows a boat across the river, her velocity relative to the water being 3 km/h due north.

a) What is her velocity relative to the Earth?

b) If the river is 250 m wide, how far east of her starting point will she reach the opposite bank?

c) How long will she take to cross the river?

35. The woman in the previous question decides, on another day, to row in such a way that the boat reaches a point on the opposite bank directly north of her starting point.

a) In what direction should she head the boat?

b) What is her velocity relative to the Earth?

c) How long will she take to cross the river?

36. An airplane pilot wishes to fly along bearing 158°. A wind of 100 km/h is blowing from the west. If the plane's airspeed is 350 km/h, find the following.

a) the plane's groundspeed

b) the heading that the pilot should take

37. Find vector equations, parametric equations, and symmetric equations of the following lines.

a) the line through the point $A(2,5,3)$ having direction numbers 2, 3, 6

b) the line through the points $(-2,-1,4)$ and $(3,2,2)$.

38. a) Determine whether the lines L_1: $\vec{r} = \overrightarrow{(1,3,0)} + t\overrightarrow{(-2,1,4)}$ and L_2: $\vec{r} = \overrightarrow{(4+3k, 2-k, -1+k)}$ are skew, or if they intersect.

b) If the lines in a) intersect, then find their point of intersection. If the lines are skew, then find the shortest distance between the lines.

39. Find a vector equation of the line that is perpendicular to the vector $\vec{u} = \overrightarrow{(1,2,3)}$ and also perpendicular to the line $\vec{r} = \overrightarrow{(0,-2,5)} + k\overrightarrow{(4,-2,1)}$ at the point corresponding to the parameter value $k = 2$.

40. Find a vector equation of the line in 2-space that is perpendicular to the line $3x + 5y + 7 = 0$, and passes through the point of intersection of the lines $\vec{r} = \overrightarrow{(4,5)} + t\overrightarrow{(1,-3)}$ and $\vec{r} = \overrightarrow{(4+2k, 3-4k)}$.

41. The line L passes through the point $A(1,3,2)$ and has direction vector $\overrightarrow{(2,1,-2)}$. Find the coordinates of two points on L that are 3 units from point A.

42. a) Find a value of c such that the lines

$$\begin{cases} x = 1 + 7t \\ y = 2 + 3t \\ z = 1 + ct \end{cases} \quad \text{and} \quad \begin{cases} x = 3 + 21k \\ y = 2 + 9k \\ z = 5 + 8k \end{cases}$$

are parallel.

b) Show that there is no value of c for which the lines in part a) intersect.

43. Determine which of the following sets of points are collinear.

a) $A(0,5,-3)$ $B(3,-1,12)$ $C(-2,9,-13)$

b) $P(5,1,9)$ $Q(-1,5,0)$ $R(8,-1,13)$

c) $L(4,2,3)$ $M(2,-4,5)$ $N(6,8,1)$

44. a) A vector through the origin makes equal angles of 60° with both the positive x-axis and positive y-axis. Find the angle the vector makes with the z-axis.

b) A vector through the origin makes equal angles of θ with the positive x-axis and with the positive y-axis. What is the smallest possible value for θ? What is the largest possible value for θ?

45. a) For what values of t, if any, will the line $\vec{r} = \overrightarrow{(3,0,5)} + k\overrightarrow{(2,t,1)}$ intersect with the line $\vec{r} = \overrightarrow{(1,2,-1)} + s\overrightarrow{(-1,-4,2)}$?

b) If a point of intersection exists in a), find its coordinates.

46. A line passes through the point $A(2,1)$ and makes an angle of 45° with the vector $\vec{u} = \overrightarrow{(3,4)}$.

a) Explain why there are two lines satisfying these conditions.

b) Find vector equations of the two lines of part a).

47. Find a vector equation of the plane that contains the point $A(-1,2,1)$ and is perpendicular to the vector $\vec{u} = \overrightarrow{(-3,0,2)}$.

48. Find a vector equation of the plane that contains the line L: $\vec{r} = \overrightarrow{(1,2,3)} + t\overrightarrow{(1,-2,2)}$ and is perpendicular to the plane $x + 2y - 3z = 8$.

49. Find the point of intersection of the plane $2x + y - z = 2$ and the line of intersection of the two planes $x - y + z = 3$ and $x + 2y - 2z = -3$.

50. Find a Cartesian equation of the plane, containing the points $T(0,1,-2)$ and $S(1,-1,3)$, that is parallel to the vector $\vec{u} = \overrightarrow{(-5,1,3)}$.

51. Find a Cartesian equation of the plane, containing the points $W(1,0,-4)$ and $N(0,2,1)$, that is perpendicular to the plane $3x + 2y - 2z = 5$.

52. Find the intersection of the line $\vec{r} = \overrightarrow{(-3 + t, -6 + 2t, 1 - t)}$ and the plane $\vec{r} = \overrightarrow{(2 + k - 3m, 1 + 2k - 3m, 1 - 3k + 2m)}$. Describe the intersection geometrically.

53. Show that the point of intersection of the line $\dfrac{x+2}{2} = 2z - 3,\ y = 3$ with the plane $x + 2y - 3z = 11$ lies on the plane $2x - y + z = 98$.

54. Given the three planes

$$\begin{aligned} x + 2y + z &= 0 \\ (k+2)x + (k+2)y + (k+1)z &= 20 \\ (k+2)x + (2k+3)y &= -20 \end{aligned}$$

a) For what values of k will the planes intersect in a point?

b) For what values of k will the planes intersect in a line?

c) For what values of k will the planes form a triangular prism?

55. Find a Cartesian equation of the plane through the points $K(3,2,1)$, $R(0,-1,1)$, and $S(-4,-3,2)$.

56. Given the point $M(1,-2,4)$.

a) Find the distance between the point M and the plane $3x + 4y - z = 5$.

b) Find the value of k so that the distance between the point M and the line $\vec{r} = \overrightarrow{(1,0,-1)} + t\overrightarrow{(3,0,k)}$ will be equal to the distance found in part a).

57. Consider the set of real numbers as the vector space $\mathbb{V}_1$ of dimension 1.

a) Show that the function defined by $f: \vec{x} \rightarrow 5\vec{x}$ is a linear transformation of $\mathbb{V}_1$.

b) Show that the function defined by $g: \vec{x} \rightarrow \overrightarrow{3x - 2}$ is *not* a linear transformation of $\mathbb{V}_1$.

(g is known as an **affine transformation** of $\mathbb{V}_1$.)

58. Determine the possible dimensions of the matrices A and B if *both* the products AB and BA are defined.

59. The transformation matrix $S = \begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix}$ is known as a **vertical shear of factor k.** The properties of a shear will appear as answers to the following questions.

a) Find the images of $(0,1)$ and $(0,b)$, where $b \in \mathbb{R}$, and describe how any point of the y-axis is transformed.

b) Find the images of $(1,0)$ and $(1,b)$, and describe how any point on the line $x = 1$ is transformed.

c) Find the image of the point $(a,0)$, where $a \in \mathbb{R}$, and describe how any point on the x-axis is transformed.

d) Find $\det(S)$, and describe the area and orientation of a figure transformed by a shear.

60. The line of equation $y = (\tan \theta)\, x$ has slope $\tan \theta$. Thus, the angle between this line and the positive x-axis is $\theta°$. Consider the transformation of matrix R that reflects the plane in the line whose equation is $y = (\tan 30°)\, x$.
   a) Find the image of $\vec{i}$ under R.
   b) Find the image of $\vec{j}$ under R.
   c) Hence write the matrix R.
   d) Write the matrix of the reflection in the line of equation $y = (\tan \theta)\, x$.

61. Given that M_ϕ represents a reflection in the line $y = (\tan \phi°)\, x$, compare M_{60} and M_{240}. Explain.

62. Given the conic C: $16x^2 + y^2 = 16$.
   a) Name the type of conic.
   b) Find, in general form, an equation for C', the image of C, under the translation $(x,y) \rightarrow (x - 3, y + 1)$.
   c) Sketch a graph of the conic C and its image C'.

63. Given the conic C: $16x^2 = y$.
   a) Name the type of conic.
   b) Find, in general form, an equation for C', the image of C, under the translation $(x,y) \rightarrow (x + 1, y + 5)$.
   c) Sketch a graph of the conic C and its image C'.

64. Given the conic C: $9x^2 - y^2 = 36$.
   a) Name the type of conic.
   b) Find, in general form, an equation for C', the image of C, under the translation $(x,y) \rightarrow (x + 2, y - 4)$.
   c) Sketch a graph of the conic C and its image C'.

65. Given the conic C: $8x^2 + y^2 + 16x + 8y + 9 = 0$.
   a) Name the type of conic.
   b) Determine the translation that changes the equation into standard form.
   c) State an equation for C', the image of C.
   d) Graph the image conic C' and then the given conic C.

66. Given the conic C: $x^2 + 4x + 8y + 12 = 0$.
   a) Name the type of conic.
   b) Determine the translation that changes the equation into standard form.
   c) State an equation for C', the image of C.
   d) Graph the image conic C' and then the given conic C.

67. Given the conic C: $4x^2 - y^2 + 32x - 4y + 64 = 0$
   a) Name the type of conic.
   b) Determine the translation that changes the equation into standard form.
   c) State an equation for C', the image of C.
   d) Graph the image conic C' and then the given conic C.

68. Given the conic C: $x^2 + 4y^2 = 16$.
   a) Find an equation for C', the image of C under a rotation of 30° about (0,0).
   b) Sketch a graph of C and C'.

69. Given the conic C: $4x^2 - 25y^2 = 100$.
   a) Find an equation for C', the image of C under a rotation of 70° about (0,0).
   b) Sketch a graph of C and C'.

70. Given the conic C: $15x^2 + 34xy + 15y^2 = 128$.
   a) Determine the type of conic.
   b) Find an angle of rotation about (0,0) that will eliminate the xy term.
   c) Find an equation of the image curve C'.
   d) Sketch the graph of the image curve C', then the graph of C, the original curve.

71. Given the conic C: $109x^2 + 236xy + 151y^2 = 1000$
   a) Determine the type of conic.
   b) Find an angle of rotation about (0,0) that will eliminate the xy term.
   c) Find an equation of the image curve C'.
   d) Sketch the graph of the image curve C', then the graph of C, the original curve.

72. Prove using mathematical induction.

a) $1^2 + 3^2 + 5^2 + \ldots + (2n-1)^2$
$= \dfrac{n(2n-1)(2n+1)}{3}$

b) $\sum_{i=1}^{n}(4i^2 - 3i + 2) = \dfrac{n(8n^2 + 3n + 7)}{6}$

73. Prove by mathematical induction that

a) $\sum_{k=1}^{n} k(k+1) = \dfrac{n(n+1)(n+2)}{3}$,
for all $n \in \mathbb{N}$.

b) Evaluate $(8)(9) + (9)(10) + (10)(11) + (11)(12) + \ldots + (98)(99)$.

74. Use mathematical induction to prove
$\left(1 + \frac{3}{1}\right)\left(1 + \frac{5}{4}\right)\left(1 + \frac{7}{9}\right)\ldots\left(1 + \frac{2n+1}{n^2}\right)$
$= (n+1)^2$

75. Prove by mathematical induction that
$\dfrac{(3n+1)7^n - 1}{9}$ is a natural number.

76. Prove that a convex polygon of n sides has
$\frac{n}{2}(n-3)$ diagonals.

77. Prove that $\dfrac{1}{n!} < \left(\dfrac{1}{2}\right)^{n-1}$ for n any natural number greater than 2.

78. Prove by induction that $x^{2n} - y^{2n}$ is divisible by $x + y$, where $n \in \mathbb{N}$.

79. Expand each of the following and simplify.

a) $(a + x)^5$

b) $(4a - 3)^4$

80. **a)** Show that the solution set of $z^6 + z^3 + 1 = 0$ is a subset of the solution set of $z^9 - 1 = 0$, and hence find the solutions of $z^6 + z^3 + 1 = 0$ in polar form.

b) Deduce the values of θ between 0 and 2π that satisfy the system of equations
$\begin{cases} \cos 6\theta + \cos 3\theta + 1 = 0 \\ \sin 6\theta + \sin 3\theta = 0 \end{cases}$

81. The equation $z^2 + (a + bi)z + (c + di) = 0$, where a, b, c, and d are non-zero real numbers, has exactly one real root. Show that $abd = b^2c + d^2$.

82. w is one of the non-real cube roots of unity.

a) Find the two possible values of the expression $1 + w + w^2$.

b) Simplify each of the expressions $(1 + w + 3w^2)^2$ and $(1 + 3w + w^2)^2$.

c) Show that the product of the two expressions in b) is 16, and that the sum is -4.

83. The matrices A and B and the non-zero vector $\vec{t}$ are given by

$$A = \begin{bmatrix} 2 & 1 \\ -7 & 1 \end{bmatrix},\ B = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix} \text{ and } \vec{t} = \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}.$$

a) i) Find the inverse of the matrix A.

ii) Use the inverse of the matrix A to solve the simultaneous equations

$$\begin{cases} 2x + y = 7 \\ -7x + y = -11, \text{ where } x, y \in \mathbb{R}. \end{cases}$$

b) Given $\vec{v} = B\vec{t}$,

i) express the components of $\vec{v}$ in terms of t_1 and t_2, and

ii) calculate the possible values of the scalar λ given that $\vec{v} = \lambda\vec{t}$ also.

c) Given $\vec{v} = B\vec{t}$ and $\vec{w} = A\vec{t}$,

i) prove that $\vec{v}$ and $\vec{w}$ are not perpendicular, and

ii) calculate the values of t_1 and t_2 when $\vec{v} - \vec{w} = \begin{bmatrix} 7 \\ 10 \end{bmatrix}$.

d) Given $\vec{t} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, calculate the components of the vector $\vec{z}$, where $\vec{z} = AB\vec{t}$.

(85 SMS)

84. The position vectors of points A, B and C are respectively $\vec{a} = \vec{i} + 2\vec{j} + 3\vec{k}$, $\vec{b} = 3\vec{i} + \vec{j} + \vec{k}$, and $\vec{c} = 2\vec{i} - \vec{j} - 2\vec{k}$ with respect to an origin O. ($\vec{i}, \vec{j}, \vec{k}$ are mutually perpendicular unit vectors.)

i) Show that O, A, B and C are coplanar by proving the linear dependence of $\vec{a}$, $\vec{b}$ and $\vec{c}$.

ii) Prove that $OABC$ is a parallelogram.

iii) Find angles ABC and ABO. Show that $OABC$ is not a rhombus.

(80 S)

85. The position vectors of four distinct points A, B, C and D are $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ respectively.

a) The mid points of $[AB]$, $[CD]$, $[BC]$, $[AD]$, $[AC]$ and $[BD]$ are E, F, G, H, L and M respectively. Find, in terms of $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$, the position vectors of the mid points of $[EF]$, $[GH]$ and $[LM]$. What does your result indicate about the lines (EF), (GH) and (LM)?

b) (AB) is perpendicular to (CD). Show that

$\vec{a} \cdot \vec{d} + \vec{b} \cdot \vec{c} = \vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{d}$

Given also that (AC) is perpendicular to (BD), prove that (AD) is perpendicular to (BC).

(87 H)

86. Two lines L_1 and L_2 have equations

$$\frac{x+3}{3} = \frac{y+4}{2} = \frac{z-6}{-2} \text{ and}$$

$$\frac{x-4}{-3} = \frac{y+7}{4} = \frac{z+3}{-1} \text{ respectively.}$$

a) Find the coordinates of a point P_1 on the line L_1 and a point P_2 on the line L_2 such that the line (P_1P_2) is perpendicular to both the lines L_1 and L_2.

b) Show that the length of $[P_1P_2]$ is 7.

c) Find the equation of the plane which contains the line L_1 and has no point of intersection with the line L_2. Give your answer in the form $ax + by + cz = d$.

(84 H)

87. i) The set of planes Π_k is given with equations
$x + 2y - 2z = 3k, k \in \mathbb{R}$.

a) Find a vector $\vec{n}$ of unit length perpendicular to the planes Π_k.

b) i) Express the equations of the planes Π_k in the form $\vec{r} \cdot \vec{n} = p$, where $\vec{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, and where the value of p is to be found in terms of k.

ii) What is the geometrical significance of

a) the magnitude of k, and

b) the sign of k?

c) Show that the perpendicular distance between the planes Π_{-2} and Π_5 is 7 units (i.e. the planes with $k = -2$ and $k = 5$ respectively).

d) Find the coordinates of the point P of intersection of the line with equation
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, y \in \mathbb{R},$$
and the plane Π_1.

e) Find the length of $[OP]$, where O is the origin, leaving your answer as an irrational number.

ii) The set of planes Π_t is given with equations
$x + ty - tz = 3, t \in \mathbb{R}$.

a) Prove that all planes of the set contain a certain line, and

b) find its equation in parametric form.

(86 S)

88. In a rectangular Cartesian coordinate system with origin O and unit basis vectors $\vec{i}$, $\vec{j}$ and $\vec{k}$, two straight lines l and l' have respective parametric equations:
$x = 6 - t,\ y = 0,\ z = t$
and $x = 0,\quad y = t',\ z = 2t'$

a) $\vec{u} = \lambda\vec{i} + \mu\vec{k}$ is the unit vector parallel to l for which $\lambda > 0$.
Find λ and μ.
Find also a unit vector $\vec{u'}$ which is parallel to l'.

b) H is a point on l with parameter t and K is a point on l' with parameter t'. If $\overrightarrow{HK}$ is perpendicular to $\vec{u}$ show that $t' = t - 3$.

c) If $\overrightarrow{HK}$ is also perpendicular to $\vec{u'}$, find the coordinates of H and K.

(81 S)

89. In a rectangular Cartesian coordinate system the points O, A, B and C have coordinates (0,0), (4,2), (−4,2) and (2,−2) respectively.

a) Prove that $|\overrightarrow{OB}| = |\overrightarrow{AC}|$.

b) i) Write in column vector form each of the vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$.

ii) Hence, given that the vector $\overrightarrow{OA} + k\overrightarrow{OB} + l\overrightarrow{OC}$ is the zero-vector, where $k, l \in \mathbb{R}$, find the values of k and l.

c) Given that $\vec{v} = \overrightarrow{OB} + n\overrightarrow{OC}$, where $n \in \mathbb{R}$.

i) If $\vec{v}$ is perpendicular to $\overrightarrow{BC}$, show that $n = 1.6$.

ii) If $\vec{v}$ is parallel to $\overrightarrow{BC}$, find the value of n.

d) It is given that matrix $M = \begin{bmatrix} -p & p \\ 0 & p \end{bmatrix}$, where $p \in \mathbb{R}$, and that $M(\overrightarrow{OA})$ means the product of M and $\overrightarrow{OA}$.

i) Given that $M(\overrightarrow{OA}) = \overrightarrow{OC}$, find the value of p.

ii) Prove that for all n and p:
$M(\overrightarrow{OA}) \neq \overrightarrow{OB} + n\overrightarrow{OC}$, where $n \in \mathbb{R}$.

(86 SMS)

90. Using a rectangular Cartesian coordinate system with origin O, a transformation $T: P \to P'$ is defined such that the coordinates of the point $P(x,y)$ are transformed to the coordinates of the point $P'(x',y')$ by means of the equation
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}.$$
   a) Find the equation of the straight line l' which is the image of the straight line l whose equation is $2x + 3y = 6$.
   b) Determine the coordinates of Q, the point of intersection of the lines l and l'.
   c) Prove that the straight line (OQ) consists of points that are their own images.
   d) Find the coordinates of the point R' that is the image of a variable point R on the line perpendicular to (OQ) through O.
   e) Give a geometrical description of the effect of the transformation T.

(85 S)

91. In a shear transformation parallel to the line with equation $y = x$, the point $A(1,1)$ is its own image and the point $B(-1,1)$ has as its image the point $B'(0,2)$. Find the (2×2) matrix that represents this shear.

(84 S)

92. For each of the following assertions concerning 2×2 matrices A and B state whether it is true or false. Prove any assertion that you consider true, and give a counterexample for any assertion you consider false.
   a) For all A and B, $(A - B)(A + 2B) = A^2 + AB - 2B^2$.
   b) For all A, $(A - I)(A + 2I) = A^2 + A - 2I$, where I is the unit 2×2 matrix.
   c) For all A and B, $(AB)^T = A^TB^T$ where A^T is the transpose of the matrix A.
   d) If $A^2 = A$ then A is a singular matrix.

(82 H)

93. The matrix M and non-zero vector $\vec{v}$ are given by
$$M = \begin{bmatrix} 1 & 4 \\ 2 & -1 \end{bmatrix} \text{ and } \vec{v} = \begin{bmatrix} x \\ y \end{bmatrix}.$$
   a) Find the components of the vector $\vec{w}$ where $\vec{w} = M\vec{v}$.
   b) It is given that $\vec{w} = \lambda\vec{v}$ where λ is a scalar, $\lambda \in \mathbb{R}$.
      i) Obtain two simultaneous equations in x and y.
      ii) Show that the condition for a non-zero vector $\vec{v}$ to exist is $\lambda^2 - 9 = 0$.
      iii) Solve the equation $\lambda^2 - 9 = 0$. For each value of λ find the corresponding set of vectors $\vec{v}$ and give a unit vector in that set.
   c) Given the matrix $N = \begin{bmatrix} 1 & 4 \\ -2 & -1 \end{bmatrix}$ find the components of the vector $\vec{u}$ where $\vec{u} = N\vec{v}$. Does there exist a value of the scalar μ, $\mu \in \mathbb{R}$, such that $\vec{u} = \mu\vec{v}$?

(83 SMS)

94. It is given that
$$M = \begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}$$
   a) Find vectors $\vec{u}$ and $\vec{v}$ such that $M\vec{u} = \vec{u}$ and $M\vec{v} = -\vec{v}$.
   b) A transformation $T: P \to P'$ is defined such that the coordinates of the point $P(x,y)$ are transformed to the coordinates of the point $P'(x',y')$ by means of the equation
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = M\begin{bmatrix} x \\ y \end{bmatrix}.$$
   By using the result of part a), or otherwise, find the equations of the lines through the origin that are invariant under the transformation T.
   c) Describe geometrically the effect of the transformation T and by applying the geometry of this result explain why $T^{-1} = T$

(84 S)

95. M is the (2×2) matrix given by

$$M = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, \ 0 \leqslant \theta < 360°$$

a) Show that $M^2 = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$.

b) Using the result of part a), or otherwise, find a (2×2) matrix X with real elements such that $X^2 + I = 0$, where I is the (2×2) unit matrix and 0 is the (2×2) zero matrix.

c) **i)** In the case when $\theta = 60°$, find the matrices M^2, M^4 and M^6, expressing your answers *exactly* in numerical form.

ii) What do each of the three matrices obtained in part c)i) represent when considered a linear transformations of a plane?

(87 S)

96. A point P is reflected in the line with equation $y = x$, to give an image point P'. P' is given a positive (anticlockwise) quarter turn about the origin to give an image point P''. Find the matrix M corresponding to the single reflection that would map P to P'' but keep the origin fixed.

(82 S)

97. Given that $A = \begin{bmatrix} 5 & 2 \\ 9 & 8 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ form the quadratic equation in λ given by $\det(A - \lambda I) = O$. Find the roots λ_1, λ_2 of this equation, where $\lambda_1 < \lambda_2$. Find a vector $\vec{e}_1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$ such that $A\vec{e}_1 = \lambda_1 \vec{e}_1$, and a vector $\vec{e}_2 = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$ such that $A\vec{e}_2 = \lambda_2 \vec{e}_2$.

Using these results

a) find the equations of two distinct lines which are mapped onto themselves by the transformation represented by the matrix A,

b) if $P = \begin{bmatrix} x_1 & x_2 \\ y_1 & y_2 \end{bmatrix}$ find $P^{-1}AP$ and deduce that $P^{-1}A^nP = \begin{bmatrix} \lambda_1^n & 0 \\ 0 & \lambda_2^n \end{bmatrix}$.

(83 H)

98. Given $z = x + iy$, $z' = x' + iy'$ and $z' = (2 + i)z$ find the (2×2) matrix M such that $\begin{bmatrix} x' \\ y' \end{bmatrix} = M \begin{bmatrix} x \\ y \end{bmatrix}$

(84 S)

99. The linear transformation L represents an anti-clockwise rotation of θ about the origin, where $\theta \in \left[0, \frac{\pi}{2}\right]$, and the linear transformation M represents a reflection in the line with equation $y = x \tan\alpha$, where $\alpha \in \left[0, \frac{\pi}{2}\right]$.

i) Show that

a) $L\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$

b) $M\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos 2\alpha \\ \sin 2\alpha \end{bmatrix}$

and find $L\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $M\begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

ii) Hence, or otherwise, find the 2×2 matrices A and B which represent the linear transformations L and M respectively.

iii) Evaluate and simplify the matrices A^2 and B^2, giving a geometrical interpretation of your results.

iv) Prove, by induction, that

$$A^n = \begin{bmatrix} \cos n\theta & -\sin n\theta \\ \sin n\theta & \cos n\theta \end{bmatrix}.$$

(85 H)

100. **a)** Show that the value of the determinant

$$\begin{vmatrix} 1 & a & 1 \\ 1 & 1 & a \\ a+1 & a & 1 \end{vmatrix} \text{ is } a(a^2 - 1).$$

b) Find such solutions as exist of the simultaneous linear equations

$$\begin{aligned} x + ay + z &= 2a, \\ x + y + az &= 0, \\ (a+1)x + ay + z &= a, \end{aligned}$$

in the cases (i) $a = 0$ and (ii) $a = -1$. Give a geometrical interpretation of your answer in each case.

(83 S)

101. a) Find, in terms of d, the value of the determinant

$$\begin{vmatrix} d & 5 & -1 \\ 1 & 3 & d \\ 1 & 4 & 7 \end{vmatrix}.$$

b) Calculate the values of d for which the value of the determinant is zero.

c) Find the solutions, if they exist, of the simultaneous linear equations.

$$\begin{aligned} dx + 5y - z &= 0, \\ x + 3y + dz &= d, \\ x + 4y + 7z &= 6, \end{aligned}$$

in each of the following cases:

i) $d = 2$;

ii) $d = 0$;

iii) $d = \dfrac{9}{2}$

(86 S)

102. A is the (2×2) matrix given by

$$A = \begin{bmatrix} 3 & 4 \\ 4 & 5 \end{bmatrix}$$

a) Write down the value of the determinant of A.

b) Write down the (2×2) matrix inverse to A.

c) Using a matrix method, solve the simultaneous equations
$3x + 4y = 12$, $4x + 5y = 20$.

d) Find the area of the closed region in the first quadrant bounded by the axes Ox, Oy and the lines l_1 and l_2 whose equations are $3x + 4y = 12$ and $4x + 5y = 20$ respectively.

e) i) Find a unit vector in the direction of each of the lines l_1 and l_2, both considered in the sense of x increasing, and

ii) hence, or otherwise, show that the value of the acute angle between these lines is 1.8°, correct to the nearest tenth of a degree

(86 S)

103. In a two dimensional rectangular Cartesian co-ordinate system the points A and B are given such that

$$\overrightarrow{OA} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \text{ and } \overrightarrow{OB} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

a) The matrix L, where $L = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, maps the points A and B onto the points C and D respectively.
Find $\overrightarrow{OC}$ and $\overrightarrow{OD}$, giving your answers in column vector form.

b) The 2×2 matrix M represents the transformation which reflects in the line $y = x$ all points of the plane.

i) Find the matrix M.

ii) Given that the transformation M maps the points C and D onto the points E and F respectively, find $\overrightarrow{OE}$ and $\overrightarrow{OF}$, giving your answers in column vector form.

iii) Given the matrix $N = M \times L$ show that $N = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$

iv) a) Find the value of det N and

b) hence, or otherwise, compare the areas of the triangles OAB and OEF.

c) The 2×2 matrix R represents the transformation which rotates through 180° about the origin O all points of the plane.

i) Find the matrix R.

ii) The matrix S is given such that $S = R \times N$.

a) Find the matrix S.

b) Find the matrix S^{-1}.

c) The transformation represented by the matrix S^{-1} maps the points A and B onto the points G and H respectively.

i) Show that $GH^2 = 2$ and

ii) Calculate the angle between $\overrightarrow{OG}$ and $\overrightarrow{OH}$, giving your answer correct to the nearest degree.

(87 SMS)

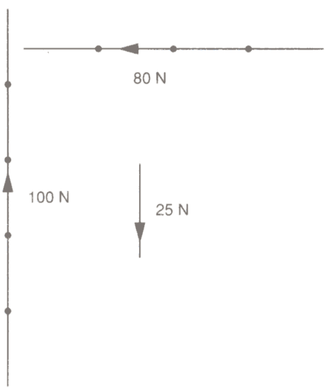
ANSWER KEY

Generally, answer is not provided where this is implied in the question.
Answers which will vary are also not provided.
Some answers have been left in unsimplified form, as a hint.
In general, numerical answers which are approximations are given with 3 significant digit accuracy, and angles to nearest degree, unless otherwise specified in the question.

Chapter One
An Introduction to Vectors

1.1 Exercises, pages 9–10

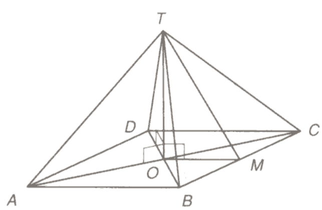
1. a) c) i) k) l)
2. a) b) ; none
3. a) $|\vec{u}| = 1$
   b) $|\vec{w}| = 3$
   c) $|\overrightarrow{AB}| = 2.5$
4. a)
   b)
   c)



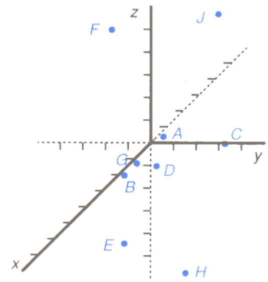
5. $AB = PQ$
6. a) Yes; equal vectors have same magnitude.
   b) No; direction could be different.
7. a) 245 km, 221°
   b) 122.5 km, 221°
8. a) no
   b) yes
   c) no
   d) no
   e) yes
   f) no
9. a) F
   b) F
   c) T
10. a) same length, same direction
    b) $\overrightarrow{PM} = \overrightarrow{MQ}$
11. $\overrightarrow{PB} = \vec{u}$, $\overrightarrow{QC} = \vec{v}$
12. a) $\overrightarrow{SR}$ d) $\overrightarrow{SP}$
    b) none e) $\overrightarrow{SI}$
    c) $\overrightarrow{IR}$ f) none
13. a) 5 b) 4 c) 3
14. $\vec{a} = \overrightarrow{BD} = \overrightarrow{EG} = \overrightarrow{CF}$
    $\vec{b} = \overrightarrow{AD} = \overrightarrow{FG} = \overrightarrow{CE}$
    $\vec{c} = \overrightarrow{BE} = \overrightarrow{DG} = \overrightarrow{AF}$
15. a) $\overrightarrow{AA'} = \overrightarrow{BB'} = \overrightarrow{CC'}$
    b) $\overrightarrow{PP'} = \overrightarrow{QQ'} = \overrightarrow{RR'} = \overrightarrow{SS'}$
    Your sketches will be of three vectors equal to
    a) $\overrightarrow{AA'}$ b) $\overrightarrow{PP'}$
16. The plane would be entirely 'shaded' in.

1.2 Exercises, page 16

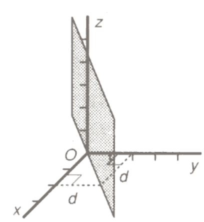
1. a) $TW \| UV \| QR \| PS$
   b) $WS \| VR \| UQ \| TP$
   c) $PR \| AB \| TV$
2. a) $TP \perp TW$ and $WS \perp TW$
   b) $TW \perp WS$ and $RS \perp WS$
   c) $TP \perp AB$ and $UQ \perp AB$
   (There are others.)
3. a) parallel
   b) skew
   c) intersecting
   d) skew
   e) intersecting
   f) skew
4. No; use a different scale, or a different direction for $\overrightarrow{QR}$, and redraw.
5.



6. $TM = 5$ cm; $TB = \sqrt{34}$ cm
7. a) TB, AC (There are others.)
   b) $ABCD$, TBC, TBA
8.



9. +++, ++−, +−+, −++, +−−, −+−, −−+, −−−
10. a) $y = z = 0$
    b) $x = z = 0$
    c) $x = y = 0$
11. a) $z = 0$
    b) $x = 0$
    c) $y = 0$
12. a plane through the origin, containing the z-axis, equidistant from the x- and y-axes



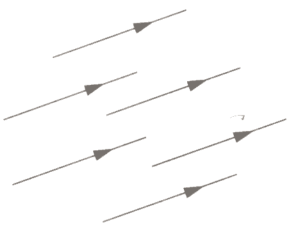
In Search of, page 20

1. a) $\alpha = 42°$
   b) $\beta = 53°$
   c) $\gamma = 26°$
2. a) $\measuredangle A = 59°$
   b) $\theta = 43°$
   c) $\phi = 53°$
3. a) 0.6 m
   b) 13°
4. a) 81.1 m
   b) 10°
5. 10.4 m; 27°; 23°

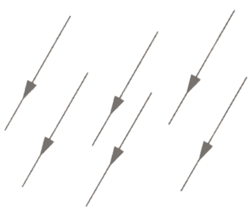
1.3 Exercises, page 25

1. $\vec{a} = \overrightarrow{(1,3)}$
   $\vec{b} = \overrightarrow{(1,0)}$
   $\vec{c} = \overrightarrow{(1,-2)}$
   $\vec{d} = \overrightarrow{(-2,-3)}$
   $\vec{s} = \overrightarrow{(0,-4)}$
   $\vec{t} = \overrightarrow{(-2,0)}$
   $\vec{u} = \overrightarrow{(-2,-2)}$
   $\vec{v} = \overrightarrow{(3,1)}$
   $\vec{w} = \overrightarrow{(-3,-1)}$

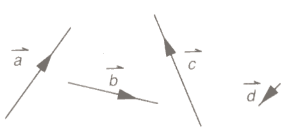
2.



3.

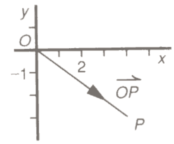


4.



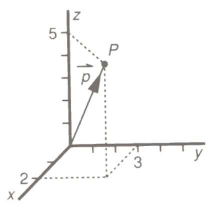
5. $|\vec{a}| = 5, |\vec{c}| = \sqrt{29},$
   $|\vec{b}| = \sqrt{17}, |\vec{d}| = \sqrt{2}$

6. a)

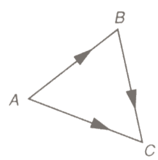


   b) $Q(-1,7)$
   c) $R(0,2,-2)$

7.



8. a)



   b) $\overrightarrow{AC} = \overrightarrow{(5,-2)}$
   c) $\overrightarrow{(x,y)} = \overrightarrow{(4,3)} + \overrightarrow{(1,-5)}$

9. $P'(5,3)$

11. $x = \sqrt{21}$

12. a) $\sqrt{13}$
    b) $\sqrt{12}$ or $2\sqrt{3}$

14. a) 700 m
    b) 500 m

15. $x = -6; w = 3$

16. $k = 7, m = 2, n = -1$

17. $h = \frac{1}{3}; k = -\frac{10}{3}$

18. a) $\overrightarrow{(3,2)}$ d) $\overrightarrow{(-4,0)}$
    b) $\overrightarrow{(-3,-2)}$ e) $\overrightarrow{(-6,-1)}$
    c) $\overrightarrow{(1,5)}$ f) $\overrightarrow{(c-a,d-b)}$

1.4 Exercises, pages 30–31

1. a) $\overrightarrow{PR}$
   b) $\overrightarrow{PR}$

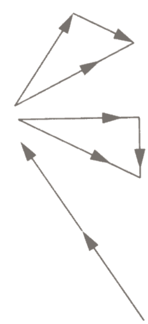
2. $\vec{u} + \vec{v} = \vec{v} + \vec{u}$

3. a) $\vec{a}$
   b) $\vec{b}$
   c) $\vec{a} + \vec{b}$
   d) $\vec{b} + \vec{a}$
   e) $\vec{c} + \vec{b}$
   f) $\vec{b} + \vec{c}$

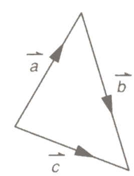
4. a) $\overrightarrow{AC}$
   b) $\overrightarrow{AC}$
   c) $\overrightarrow{BG}$
   d) $\overrightarrow{AG}$
   e) $\overrightarrow{DE}$
   f) $\overrightarrow{EF}$
   (There are others.)

5. a) $\vec{u} + \vec{w}$
   b) $\vec{u} + \vec{w}$
   c) $\vec{v} + \vec{w}$
   d) $\vec{u} + \vec{v} + \vec{w}$

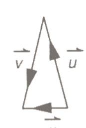
6. a) b) c)



7.



8. a) $\vec{w} = \overrightarrow{(-2,0)}$
   b)



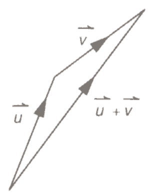
1.4 Exercises, pages 30–31, continued

9. a) $\overrightarrow{(1,8)}$
   b) $\overrightarrow{(5,-1)}$
   c) $\overrightarrow{(2,1)}$
   d) $\overrightarrow{(2,1)}$

10. a) $\overrightarrow{(3,-2,5)}$
    b) $\overrightarrow{(-4,-4,-1)}$
    c) $\overrightarrow{(-1,-8,8)}$
    d) $\overrightarrow{(-1,-8,8)}$

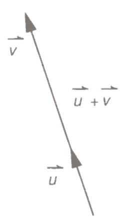
11. Vector addition is commutative.

12. a) $\overrightarrow{(6,8)}$
    b)



c) $|\vec{u}| = \sqrt{29}; |\vec{v}| = 5;$
   $|\vec{u} + \vec{v}| = 10$
d) no (triangle inequality)

13. a) $\overrightarrow{(-3,9)}$
    b)



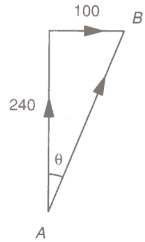
c) $|\vec{u}| = \sqrt{10}; |\vec{v}| = 2\sqrt{10};$
   $|\vec{u} + \vec{v}| = 3\sqrt{10}$
d) $|\vec{u} + \vec{v}| = |\vec{u}| + |\vec{v}|$
   (vectors in a line)

14. a) $\overrightarrow{(7,6)}$
    b) $\overrightarrow{(7,6)}$
    $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w});$
    Vector addition is associative.

15. $p = 4, q = -9$

16. $\vec{a} = \overrightarrow{(1,0,0)}$

17.



$|\overrightarrow{AB}| = 260$; bearing 023°

18. 8.26; bearing 226°

19. a) $|\vec{p} + \vec{q}| = 6.61$
    b) $\theta = 25°$

20. a) $4\sqrt{10}$ or 12.6
    b) 13

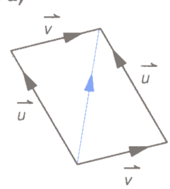
21. $12.6 < 12 + 4;$
    A, B, C collinear

1.5 Exercises, page 37

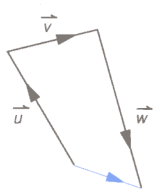
1. a) $\overrightarrow{AC}$
   b) $\overrightarrow{AC}$

2. a) $\overrightarrow{AA}$
   b) $\overrightarrow{AC}$
   c) $\overrightarrow{AD}$
   d) $\overrightarrow{AE}$
   e) $\overrightarrow{AC}$

3. a) $\vec{u} + \vec{v} = \vec{v} + \vec{u} = \overrightarrow{(1,6)}$
   c) as a)
   b)



4. b)



a) c) $(\vec{u} + \vec{v}) + \vec{w} =$
$\vec{u} + (\vec{v} + \vec{w}) = \overrightarrow{(3,-1)}$

5.

	commutative	associative
+	yes	yes
×	yes	yes
−	no; $8 - 3 \neq 3 - 8$	no; $(8 - 3) - 1 \neq 8 - (3 - 1)$
÷	no; $2 \div 8 \neq 8 \div 2$	no; $2 \div (8 \div 4) \neq (2 \div 8) \div 4$

6. a) no b) $(2^3)^4 \neq 2^{3^4}$

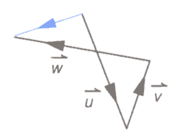
7. a) $\overrightarrow{AC}$
   b) $\overrightarrow{XZ}$
   c) $\overrightarrow{AB}$
   d) $\overrightarrow{QQ}$

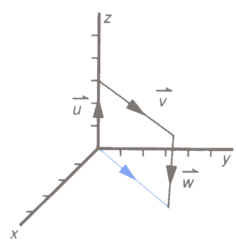
8. a) A diagram gives sufficient explanation.
   b) $\overrightarrow{PP}$
   c) 0
   d) no

9. a) $\sqrt{38}$
   b) $\sqrt{38}$
   c) $\sqrt{38}$
   d) 5

10. a) $\overrightarrow{(-3,-1)}$
    b) $\overrightarrow{(5,5,0)}$

11. a)





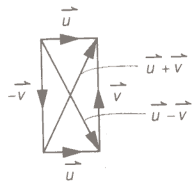
b) neither

12. a) $\sqrt{89}$
    b) $\sqrt{105}$
    c) 23°

1.6 Exercises, page 42

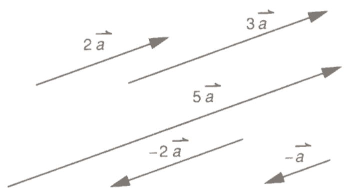
1. a) F b) T c) T d) F
2. $\overrightarrow{RS} = \overrightarrow{QS} - \overrightarrow{QR}$
3. a) $\overrightarrow{RP}$
   b) $\overrightarrow{XZ}$
   c) $\overrightarrow{AB}$
   d) $\vec{0}$
   e) $\overrightarrow{AC}$
4. a) $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$
   b) $\overrightarrow{QR} = \overrightarrow{OR} - \overrightarrow{OQ}$
   c) $\overrightarrow{RS} = \overrightarrow{OS} - \overrightarrow{OR}$
   d) $\overrightarrow{RP} = \overrightarrow{OP} - \overrightarrow{OR}$
5. $\vec{d} = \vec{a} + \vec{c} - \vec{b}$
7. a) $\overrightarrow{(3,-2)}$
   b) $\overrightarrow{(-4,9)}$
   c) $\overrightarrow{(1,-7)}$
   d) $\overrightarrow{(-1,7)}$
8. a) $\overrightarrow{(-3,4,-9)}$
   b) $\overrightarrow{(7,2,6)}$
   c) $\overrightarrow{(-4,-6,3)}$
   d) $\overrightarrow{(4,6,-3)}$
9. $(\vec{r} - \vec{p}) = -(\vec{p} - \vec{r})$
10. a) $\overrightarrow{(-2,12)}$
    b) $\overrightarrow{(6,-6)}$
    c) $\overrightarrow{(-7,-1,-8)}$
    d) $\overrightarrow{(1,7,-6)}$
11. $m = 6, n = -2, k = 9$
12. a) $\overrightarrow{(5,1)}$
    b) $\overrightarrow{(-8,-3)}$
    c) $\overrightarrow{(3,2)}$
13. $\vec{0}$ (by vector addition)
14. a) $\overrightarrow{(1,2,16)}$
    b) $\overrightarrow{(-5,-6,-3)}$
    c) $\overrightarrow{(4,4,-13)}$
15. $(3,1)$
16. $(-4,-4,-4)$
17. $(-3,-2)$

18. a) $|\vec{u} + \vec{v}| = \sqrt{|\vec{u}|^2 + |\vec{v}|^2}$
    b)

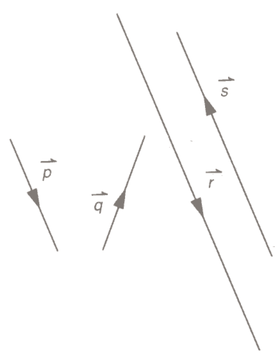


1.7 Exercises, pages 46-47

1.

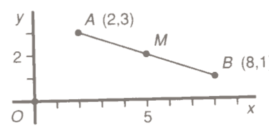


2. a) b)
3. a) $2\vec{u} + \vec{v}$
   b) $2\vec{v}$
   c) $2\vec{v}$
   d) $2\vec{u} - 2\vec{v}$
   e) $-2\vec{u} - 2\vec{v}$
4.



$\vec{p} \| \vec{r} \| \vec{s}$
$\vec{p}$ and $\vec{r}$ in same direction

5. a) $\overrightarrow{BC} = 2\overrightarrow{AB}$
   b) $\overrightarrow{AB} = \frac{1}{2}\overrightarrow{BC}$
   c) same slope
6. a) $\overrightarrow{BA} = \overrightarrow{(-2,3)}$
   b) $\overrightarrow{AC} = \overrightarrow{(6,-9)}$
   c) $\overrightarrow{CA} = \overrightarrow{(-6,9)}$
7. a) $\overrightarrow{(9,0)}$
   b) $\overrightarrow{(-3,-12,21)}$
   c) $\overrightarrow{\left(\frac{17}{2}, \frac{3}{2}, 3\right)}$
8. a) $\overrightarrow{(10,5,-15)}$
   b) $\vec{0}$
   c) coplanar
9. a) $5\sqrt{14}$
   b) 0
10. Answers will vary.
11. a) $\vec{v}$
    b) $2\vec{u}$
    c) $-\vec{v}$
    d) $\vec{0}$
12. a) $\overrightarrow{AB}$
    b) $\vec{0}$
    c) $\overrightarrow{BA}$
15. 1
16. a) $\overrightarrow{(5,2)}$
    b) c)



17. a) $\overrightarrow{MA} = -\overrightarrow{MB}$
    b) M midpoint of AB
    c) $\overrightarrow{OM} = \frac{1}{2}\overrightarrow{OA} + \frac{1}{2}\overrightarrow{OB}$
18. a) 16
    b) 16
    c) 5
    d) 8.73

1.8 Exercises, page 51

1. $\overrightarrow{AB} = \vec{u}$; $\overrightarrow{CB} = 2\vec{v} - \vec{u}$;
   parallelogram
2. rhombus
3. $\overrightarrow{AB} = \frac{1}{2}\vec{c}$; $\overrightarrow{CB} = \vec{a} - \frac{1}{2}\vec{c}$
   trapezoid

1.8 Exercises, page 51, continued

4. a) $\overrightarrow{OS} = 2\vec{p}$
$\overrightarrow{OT} = 4\vec{q}$
$\overrightarrow{QP} = \vec{p} - \vec{q}$
$\overrightarrow{TS} = 2\vec{p} - 4\vec{q}$
$\overrightarrow{QR} = 3(\vec{p} - \vec{q})$
$\overrightarrow{OR} = 3\vec{p} - 2\vec{q}$
$\overrightarrow{TR} = 3\vec{p} - 6\vec{q}$
b) $\overrightarrow{TR} = \frac{3}{2}\overrightarrow{TS}$

12. c) parallelogram

1.9 Exercises, page 56

1. a) $\overrightarrow{(1,1)}$
b) $\overrightarrow{(-5,1)}$
c) $\overrightarrow{(0,-2)}$

2. a) $\vec{u} = 2\vec{i} - 7\vec{j}$
b) $\vec{v} = 6\vec{i} + \vec{j}$
c) $\vec{w} = -3\vec{i}$

3. a) $\overrightarrow{(1,2,3)}$
b) $\overrightarrow{(4,0,-1)}$
c) $\overrightarrow{(0,-1,-1)}$

4. a) $\vec{u} = 2\vec{i} - 4\vec{j} + 6\vec{k}$
b) $\vec{v} = -\vec{j} - \vec{k}$
c) $\vec{w} = 10\vec{j}$

5. a) $18\vec{i} - 6\vec{j}$
b) $2\vec{i} - 2\vec{j} + 2\vec{k}$
c) $15\vec{i} - 5\vec{k}$
d) $2\vec{i} + 4\vec{j}$

6. a) $-\vec{i} + 8\vec{j}$
b) $-2\vec{i} - 11\vec{k}$
c) $-8\vec{k}$

7. $\vec{b}, \vec{c}, \vec{d}, \vec{f}$

8. $\vec{e}_u = \frac{1}{5}\overrightarrow{(3,-4)}$
$\vec{e}_v = \overrightarrow{(1,0,0)}$
$\vec{e}_w = \frac{1}{\sqrt{41}}\overrightarrow{(2,6,-1)}$
$\vec{e}_z = \frac{1}{\sqrt{3}}\overrightarrow{(1,1,1)}$
$\vec{e}_p = \frac{1}{4}\overrightarrow{(\sqrt{7},3)}$
$\vec{e}_q = \frac{1}{\sqrt{41}}\overrightarrow{(5,-4)}$

10. a) $\overrightarrow{PQ} = \overrightarrow{(7,-5)}$
$\vec{e}_{PQ} = \frac{1}{\sqrt{74}}\overrightarrow{(7,-5)}$
b) $\overrightarrow{PQ} = \overrightarrow{(-1,2,-5)}$
$\vec{e}_{PQ} = \frac{1}{\sqrt{30}}\overrightarrow{(-1,2,-5)}$

11. $\vec{e}_u = \frac{1}{13}\overrightarrow{(-5,12)}$
$\vec{e}_v = \frac{2}{13}\overrightarrow{\left(\frac{3}{2},2,-6\right)}$
$\vec{e}_w = \frac{1}{\sqrt{53}}\overrightarrow{(2,0,7)}$

12. a) $\sqrt{2}$ b) $\sqrt{5}$

13. a) $\frac{1}{\sqrt{33}}\overrightarrow{(4,4,1)}$
b) $\frac{1}{15}\overrightarrow{(12,0,-9)}$
c) $\frac{1}{2}\overrightarrow{(0,0,-2)}$

15. $\overrightarrow{AB} = -\vec{i} + 4\vec{j}$

In Search of, page 59

2. a) triangular shapes
b) Canadian families
c) the school's classes
d) directions (pointing either way)
e) lengths
f) integers, e.g. $(4,1) \in 3$ but $(1,4) \notin -3$
g) rational numbers, e.g. $(4,6) \in \frac{2}{3}$

Inventory, page 62

1. direction
2. vector
3. scalar
4. directed; ordered pair; ordered triple
5. length (or magnitude); direction
6. -3; 2
7. points; non-skew
8. verticals; parallels
9. intersecting; parallel
10. $\overrightarrow{(2,-3,4)}$
11. $|\vec{v}|$; 3
12. $\overrightarrow{(0,2)}$
13. $\overrightarrow{FH}$
14. $\overrightarrow{LK}$
15. $\overrightarrow{(10,5,20)}$
16. parallel; $|k||\vec{u}|$
17. $\overrightarrow{PQ} = \overrightarrow{(-2,-2)}$
18. unit
19. $4\vec{i} - 3\vec{j}$
20. unit; $\frac{1}{5}\overrightarrow{(4,-3)}$

Review Exercises, page 63–65

1. a) equal lengths $AP = PM$; same direction
b) $\overrightarrow{AP} = \overrightarrow{PM} = \overrightarrow{MQ} = \overrightarrow{QB}$
c) $\overrightarrow{BM} = \overrightarrow{MA} = \overrightarrow{QP}$

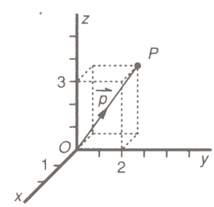
2. a) $\overrightarrow{PB} = \vec{u}$
b) $\overrightarrow{RC} = \vec{u}$
c) $\overrightarrow{DR} = \vec{u}$
d) $\overrightarrow{QC} = \vec{v}$
e) $\overrightarrow{AS} = \vec{v}$

3. c) $\measuredangle DON$, $\measuredangle DOM$, $\measuredangle DOP$

4. a) 60°
b) 55°

5. a) DB and AC (There are others.)
b) DAB, DBC, DCA

6.

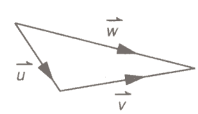


7. $P'(6,0,-1)$

8. $\pm\frac{\sqrt{11}}{6}$

9. $\pm\sqrt{122}$

10. a) $\vec{w} = \overrightarrow{(8,-2)}$
b)

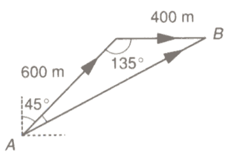


c) $|\vec{w}| = 2\sqrt{17}$

11. a) $\vec{u}$
b) $\vec{v}$
c) $\vec{v} + \vec{w} + \vec{z}$
d) $\vec{v} + \vec{u}$
e) $\vec{u} + \vec{v} + \vec{w} + \vec{z}$

12. $k = -4, n = 0, m = 6$

13.



$|\overrightarrow{AB}| = 927$ m, bearing 063°

14. a) $\overrightarrow{RP}$
b) $\vec{0}$

15. $\overrightarrow{LM} = \overrightarrow{(-13,5,7)}$

16. a) $\overrightarrow{AC} - \overrightarrow{AB}$
b) $\overrightarrow{AD} - \overrightarrow{AC}$
c) $-\overrightarrow{AB}$

17. $\vec{s} - \vec{p} - \vec{r}$

18. a) $\overrightarrow{(3,-6,-7)}$
b) $\overrightarrow{(3,-2,9)}$
c) $\overrightarrow{(-6,8,-2)}$
d) $\overrightarrow{(4,-12,-18)}$
e) $\overrightarrow{(6,0,-10)}$
f) $\overrightarrow{\left(\frac{5}{2},-7,\frac{9}{2}\right)}$

19. a) $\overrightarrow{QR} = 2\overrightarrow{PQ}$
b) collinear

20. $P(3,0,-1)$
$Q(7,-1,1)$
$R(15,-3,5)$

21. a) 12.5
b) 10
c) $\sqrt{29}$

22. a) $\overrightarrow{DC}$
b) $\overrightarrow{DB}$
c) $\vec{0}$
d) $\overrightarrow{DC}$
e) $\overrightarrow{AD}$

25. a) $-15\vec{i} + 24\vec{j}$
b) $32\vec{i} + 16\vec{j} - 47\vec{k}$

26. a) $\frac{1}{\sqrt{29}}\overrightarrow{(2,5)}$
b) $\frac{1}{3\sqrt{2}}\overrightarrow{(1,-4,1)}$
c) $\frac{1}{K}\overrightarrow{(d-a, e-b, f-c)}$, where
$K = \sqrt{(d-a)^2 + (e-b)^2 + (f-c)^2}$

27. a) $\sqrt{2}$
b) $\sqrt{3}$
c) $\sqrt{10}$
d) $\sqrt{14}$

29. E

30. a) $\overrightarrow{OP} = \begin{bmatrix} 6 \\ 0 \\ 3 \end{bmatrix} \overrightarrow{OQ} = \begin{bmatrix} 3 \\ 6 \\ 6 \end{bmatrix}$
b) $OP^2 = 45, OQ^2 = 81$
$OQ^2 \neq OP^2 + PQ^2$
c) $RS = \frac{3\sqrt{14}}{2}$
d) $6\sqrt{2}$

Chapter Two
Linear Dependence

2.1 Exercises, pages 71-72

1. a) $\vec{a} \| \vec{b}$
   b) $\vec{a} = s\vec{b}$, $m\vec{a} + k\vec{b} = \vec{0}$
   c) $s \in \mathbb{R}$, m, k not both zero
2. $\vec{x} = s\vec{y}$ or $m\vec{x} + k\vec{y} = \vec{0}$
3. The line segments can be parallel and distinct.
4. $\vec{z}$ and $\vec{d}$ are linearly dependent.
5. $\vec{m} \| \vec{k}$
6. a) $\vec{a} \| \overrightarrow{PR}$
   b) $\vec{a} \nparallel \overrightarrow{PT}$
7. $\vec{u}, \vec{w}, \vec{r}\ \vec{t}$
8. $\vec{a} \| \vec{c} \| \vec{u} \| \vec{w} \| \vec{r} \| \vec{t}$
   $\vec{b} \| \vec{v}$
9. a) $\overrightarrow{(4,6)}, \overrightarrow{(6,9)}, \overrightarrow{(1,1.5)}$
   b) same as a)
   c) $\overrightarrow{(1,3)}$
10. a) $\overrightarrow{(8,2,6)}, \overrightarrow{(12,3,9)}, \overrightarrow{(2,0.5,1.5)}$
    b) same as a)
    c) $\overrightarrow{(0,1,3)}$
11. $\vec{c}, \vec{e}$
12. a) $\vec{a} \nparallel \vec{b}$
    b) yes
13. a) $r = v = 0$
    b) no
14. a) c) d) e)
15. a) $\overrightarrow{DC}$
    b) $\overrightarrow{AE}, \overrightarrow{EC}$
    c) $\overrightarrow{EC}, \overrightarrow{AC}$
    d) $\overrightarrow{BE}, \overrightarrow{ED}$
16. None are parallel.
17. a) $\overrightarrow{PQ} = \overrightarrow{(-1,3)}$
       $\overrightarrow{PR} = \overrightarrow{(-2,6)}$
    b) $2\overrightarrow{PQ} = \overrightarrow{PR}$
    c) collinear
18. a) $\overrightarrow{AB} = \overrightarrow{(4,2,6)}$
       $\overrightarrow{AC} = \overrightarrow{(-2,-1,-3)}$
    b) $\overrightarrow{AB} = -2\overrightarrow{AC}$
    c) collinear
19. a) $m = k = 0$; no
    b) if $\vec{a} = \vec{b}$, then $m = -k$
       if $\vec{a} = -\vec{b}$, then $m = k$;
       yes
20. a) $s = t = 0$
    b) $r = 0, m = 3$
    c) $x = 1, y = -2$
    d) $z = 3, k = -\frac{7}{3}$
21. $m = 3, k = -5$
22. $d = -2, c = -\frac{4}{5}$
23. $\vec{f} = 2\vec{a}$
26. Use $\overrightarrow{AC} = 3\overrightarrow{AB}$

2.2 Exercises, page 80

1. a) coplanar
   b) $\vec{a} = s\vec{b} + r\vec{c}$;
      $m\vec{a} + k\vec{b} + t\vec{c} = \vec{0}$
   c) m, k, t not all zero
2. $k\vec{x} + m\vec{y} + t\vec{z} = \vec{0}$
   $\vec{x} = a\vec{y} + b\vec{z}$
3. A vector can be represented by many parallel line segments.
4. a) coplanar
   b) not all zero
8. a) $\vec{c} = 3\vec{a} - 2\vec{b}$
   b) $\vec{f} = -\vec{d} - 3\vec{e}$
   c) $\vec{h} = -2\vec{g} + 5\vec{n}$
   d) $\vec{p} = 0\vec{q} - 2\vec{r}$
9. 17
10. 0
11. a) $\vec{a} = 3\vec{u}$
    b) impossible
    c) $\vec{a} = 3\vec{u} + 0\vec{v}$
    d) $\vec{a} = 3\vec{u} + 0\vec{v} + 0\vec{w}$
12. a) impossible
    b) impossible
    c) $\vec{a} = 2\vec{u} - \vec{v}$
    d) $\vec{a} = 2\vec{u} - \vec{v} + 0\vec{w}$
13. a) $\vec{a} = 0\vec{u} + 3\vec{v}$
    b) $\vec{a} = 0\vec{u} + 3\vec{v} + 0\vec{w}$
    c) $\vec{a} = 0\vec{u} + 3\vec{v} + 0\vec{w} + 0\vec{t}$
14. a) impossible
    b) $\vec{a} = \frac{7}{3}\vec{u} + \vec{v} + \frac{1}{3}\vec{w}$
    c) $\vec{a} = \frac{7}{3}\vec{u} + \vec{v} + \frac{1}{3}\vec{w} + 0\vec{t}$
16. $m = 2, k = 0, p = 1$

2.3 Exercises, page 87

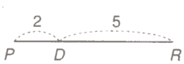
1. $w = x = z = 0$
2. $\vec{a}, \vec{b}, \vec{d}$
4. $\vec{u} = \frac{1}{2}\vec{v} + \frac{1}{2}\vec{w}$
6. a) dependent, coplanar
   b) independent, not coplanar
   c) independent, not coplanar
   d) dependent, coplanar
   e) dependent, coplanar
7. b) $\vec{c} = 2\vec{a} + \vec{b}$,
      $\vec{d} = 5\vec{a} - 4\vec{b}$,
      $\vec{e} = 3\vec{a} + 0\vec{b}$
8. a) b)
9. b) $\vec{d} = 2\vec{a} + \vec{b} - 3\vec{c}$
      $\vec{e} = 0\vec{a} - 3\vec{b} + 4\vec{c}$
      $\vec{f} = \vec{a} - \vec{b} + 2\vec{c}$
12. a) $w = 3$
    b) $w \neq 3$
13. a) $m = 1$ or $m = -1$
    b) $m \neq 1$ and $m \neq -1$
14. a) At least one is a linear combination of the other three.

Making Connections, page 89

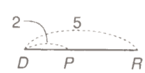
3. 5
4. 366 (377 in leap year)
5. 1096 (1099 in leap year)

2.4 Exercises, page 94

1. a)



   b)



2. a) b) $\overrightarrow{OD} = \frac{3}{7}\overrightarrow{OP} + \frac{4}{7}\overrightarrow{OR}$

3. a) $\frac{5}{8}\overrightarrow{OP} + \frac{3}{8}\overrightarrow{OR}$
   b) $\frac{1}{5}\overrightarrow{OP} + \frac{4}{5}\overrightarrow{OR}$
   c) $\frac{2}{9}\overrightarrow{OP} + \frac{7}{9}\overrightarrow{OR}$
   d) $\frac{11}{17}\overrightarrow{OP} + \frac{6}{17}\overrightarrow{OR}$
   e) $\frac{1}{2}\overrightarrow{OP} + \frac{1}{2}\overrightarrow{OR}$
4. a) b) $\overrightarrow{OD} = -3\overrightarrow{OP} + 4\overrightarrow{OR}$
5. a) $\frac{5}{2}\overrightarrow{OP} - \frac{3}{2}\overrightarrow{OR}$
   b) $-\frac{1}{3}\overrightarrow{OP} + \frac{4}{3}\overrightarrow{OR}$
   c) $-\frac{2}{5}\overrightarrow{OP} + \frac{7}{5}\overrightarrow{OR}$
   d) $\frac{11}{5}\overrightarrow{OP} - \frac{6}{5}\overrightarrow{OR}$
   e) $2\overrightarrow{OP} - \overrightarrow{OR}$
6. $\frac{12}{5}\overrightarrow{OP} - \frac{7}{5}\overrightarrow{OQ}$
7. a) between PR, closer to R
   b) outside PR, closer to R
   c) $D = R$
8. $\left(\frac{37}{10}, \frac{71}{10}\right)$
9. a) $\left(\frac{27}{7}, \frac{50}{7}\right)$
   b) $\left(\frac{14}{13}, \frac{83}{13}\right)$
   c) $\left(\frac{3}{2}, \frac{13}{2}\right)$
   d) $\left(-\frac{65}{8}, \frac{31}{8}\right)$
   e) $\left(\frac{89}{8}, \frac{73}{8}\right)$
   f) (18,11)
10. $\left(\frac{23}{6}, \frac{11}{3}, \frac{11}{6}\right)$
11. a) $\left(\frac{31}{7}, \frac{34}{7}, \frac{17}{7}\right)$
    b) $\left(\frac{33}{18}, \frac{17}{4}, \frac{17}{8}\right)$
    c) (4,4,2)
    d) $\left(\frac{12}{5}, \frac{4}{5}, \frac{2}{5}\right)$
    e) (9,14,7)
    f) (7,10,5)
12. a) $\frac{6}{11}$
    b) 5 : 6
13. $\overrightarrow{OT} = \frac{1}{5}\overrightarrow{OA} + \frac{4}{5}\overrightarrow{OR}$
14. a) $\overrightarrow{(3,-1)}, \overrightarrow{(5,6)}, \overrightarrow{(1,1)}$
    b) (3,2)
    c) (3,2)
    d) (3,2)
    e) concurrent; divide each other in ratio 2 : 1

In Search of, page 96

a) (1,2,3)
b) (3,2,−2)
c) $\left(\frac{27-7t}{13}, \frac{34+9t}{13}, t\right), t \in \mathbb{R}$
d) no solution

2.5 Exercises, page 101

1. a) Any two of $\overrightarrow{PQ}, \overrightarrow{PR}, \overrightarrow{QR}$ are linearly dependent.
   b) Any two of $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{AC}$ are linearly dependent.
2. $\overrightarrow{AB} = -2\overrightarrow{AC}$
4. $\overrightarrow{PR} = -2\overrightarrow{PQ}$
6. a) collinear
   b) not collinear
   c) not collinear
   d) collinear
7. Scalars sum to 1.
   a) 2 : 1
   b) 3 : 4
   c) 4 : −1
   d) −7 : 8
8. $\frac{1}{3}$
9. a) $m = 5$
   b) $n = \frac{2}{9}$
   c) $s = -\frac{2}{3}$
10. a) $\overrightarrow{PD}, \overrightarrow{PR}, \overrightarrow{PS}$ are linearly dependent.
    b) $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$ are linearly dependent.
11. $\overrightarrow{PS} = 2\overrightarrow{PQ} + 3\overrightarrow{PR}$
13. a) coplanar; $\overrightarrow{PS} = 3\overrightarrow{PQ} - 2\overrightarrow{PR}$
    b) coplanar; $\overrightarrow{WZ} = \overrightarrow{WX} + 2\overrightarrow{WY}$
    c) not coplanar
    d) coplanar; $\overrightarrow{KN} = -2\overrightarrow{KM} + 0\overrightarrow{KL}$
14. $k\overrightarrow{AB} + p\overrightarrow{AC} = \vec{0}$
15. a) $\overrightarrow{AB} = \vec{b} - \vec{a}, \overrightarrow{AC} = \vec{c} - \vec{a}$, $\overrightarrow{AZ} = -6\vec{a} + 2\vec{b} + 3\vec{c}$
    b) $\overrightarrow{AZ} = 2\overrightarrow{AB} + 3\overrightarrow{AC}$

2.6 Exercises, page 105

1. 2 : 1
2. 7 : 2
3. 2 : 1
4. 1 : 1
5. 1 : 2
6. 1 : k
7. 5 : 2; 6 : 1
8. 5 : 7
9. 2 : 3
10. 1 : k + 1
11. 1 : 2

Inventory, page 108

1. collinear; $k\vec{a}$; $t\vec{b}$
2. coplanar; combination; $s\vec{a} + t\vec{b}$; zero; $m\vec{a} + k\vec{b} + p\vec{c} = \vec{0}$
3. independent
4. independent
5. a) 0; 0
   b) 0; 0; 0
6. a) $\overrightarrow{(6,10)}$
   b) $\overrightarrow{(3,0)}$
   c) $\overrightarrow{(4,6,2)}$
   d) $\overrightarrow{(2,3,0)}$
   Answers may vary.
7. 0; $2m + 4k = 0$
8. linearly dependent
9. linearly dependent
10. a) $\frac{7}{12}, \frac{5}{12}$
    b) $\frac{7}{2}, -\frac{5}{2}$
11. a) collinear
    b) coplanar
12. linearly independent
13. two; independent
14. three; independent

Review Exercises, pages 109–111

1. a) parallel
   b) $\vec{a} = k\vec{b}$; $m\vec{a} + t\vec{b} = \vec{0}$
   c) $k \in \mathbb{R}$;
   not both m, t can be zero,
   $m, t \in \mathbb{R}$
2. a) coplanar
   b) $\vec{a} = k\vec{b} + m\vec{c}$;
   $s\vec{a} + t\vec{b} + p\vec{c} = \vec{0}$
   c) $k, m, s, t, p \in \mathbb{R}$;
   not all of s, t, p can be zero.
3. $a = b = 0$
4. $s = t = r = 0$
5. $2\vec{a}, -8\vec{b}, -7\vec{d}$
6. a) $\vec{x} \nparallel \vec{y}, \vec{x} \nparallel \vec{z}, \vec{x} \nparallel \vec{w}$,
   b) $\vec{q}, \vec{y}, \vec{z}$
7. $\vec{a}, \vec{b}, \vec{d}$
8. $\vec{z}, \vec{h}, \vec{w}$
9. a) $\overrightarrow{(-3,-5)}, \overrightarrow{(6,10)}\ \overrightarrow{(1.5,2.5)}$
   b) $\overrightarrow{(3,0)}$
   Answers may vary.
10. a) $\overrightarrow{(-7,-1,2)}, \overrightarrow{(14,2,-4)}$
    $\overrightarrow{(21,3,-6)}$
    b) $\overrightarrow{(7,1,0)}, \overrightarrow{(7,0,-2)}$
    Answers may vary.
12. b) $\overrightarrow{(-3,2)} = -2\overrightarrow{(1.5,-1)}$
    c) $\overrightarrow{(9,3,6)} = 3\overrightarrow{(3,1,2)}$
13. $3\vec{a} + 4\vec{b}$
14. $2\vec{a} - \vec{b} + 3\vec{c}$
15. not coplanar
16. yes
18. a) $\vec{a}, \vec{b}$ are linearly independent
    b) $\vec{a}, \vec{b}, \vec{c}$ are linearly independent
19. a) linearly independent, not coplanar
    b) linearly dependent, coplanar
    c) linearly independent, not coplanar
20. b) $4\vec{a} + 2\vec{b}$
21. b) $3\vec{a} + 4\vec{b} + \frac{1}{2}\vec{c}$
22. $k = 3$ $(\vec{c} = -2\vec{a} + 3\vec{b})$
23. a) $\frac{5}{16}\overrightarrow{OP} + \frac{11}{16}\overrightarrow{OR}$
    b) $\frac{7}{12}\overrightarrow{OP} + \frac{5}{12}\overrightarrow{OR}$
    c) $\frac{1}{8}\overrightarrow{OP} + \frac{7}{8}\overrightarrow{OR}$
    d) $\frac{11}{17}\overrightarrow{OP} + \frac{6}{17}\overrightarrow{OR}$
24. a) $\frac{5}{4}\overrightarrow{OA} - \frac{1}{4}\overrightarrow{OB}$
    b) $-\frac{3}{4}\overrightarrow{OA} + \frac{7}{4}\overrightarrow{OB}$
    c) $\frac{9}{5}\overrightarrow{OA} - \frac{4}{5}\overrightarrow{OB}$
25. $2\overrightarrow{OP} - \overrightarrow{OQ}$
26. a) $(-2.5, 2, 5.75)$
    b) $\left(10, \frac{31}{3}, 12\right)$
27. $\overrightarrow{OT} = -\frac{13}{11}\overrightarrow{OA} + \frac{24}{11}\overrightarrow{OR}$
28. a) $\overrightarrow{PQ} = k\overrightarrow{PR}$
    b) $\overrightarrow{AB} = m\overrightarrow{AC} + t\overrightarrow{AD}$
29. a) collinear, $\overrightarrow{AC} = 3\overrightarrow{AB}$
    b) not collinear
30. a) coplanar, $\overrightarrow{AB} = 3\overrightarrow{AC} - 2\overrightarrow{AD}$
    b) not coplanar
31. $2:5$
32. $k = 3, m = -1$
34. a) $\overrightarrow{AB} = \vec{b} - \vec{a}$,
    $\overrightarrow{AC} = \vec{c} - \vec{a}$,
    $\overrightarrow{AZ} = 3\vec{a} + 4\vec{b} - 7\vec{c}$
    b) $\overrightarrow{AZ} = 4\overrightarrow{AB} - 7\overrightarrow{AC}$
35. $11:4$
36. $5:6$
37. $3:2$
38. $3:2$; $2:1$
39. $7:6$
41. a) $\overrightarrow{AB} = \vec{b} - \vec{a}$
    $\overrightarrow{AC} = \vec{c} - \vec{a}$
    $\overrightarrow{AZ} = -5\vec{a} + 2\vec{b} + 3\vec{c}$
    b) $\overrightarrow{AZ} = 2\overrightarrow{AB} + 3\overrightarrow{AC}$
    c) coplanar
42. $\frac{11}{3}\vec{i} + 2\vec{j} - \frac{13}{3}\vec{k}$
43. $\frac{2}{5}$
44. A
45. D

Chapter Three The Multiplication of Vectors

3.1 Exercises, page 120

1. a) $56°$
   b) $0°$
   c) $145°$
2. a) 3.830; 3.213
   b) −2.394; 6.578
   c) 0, 13
3. a) 1.50; $1.50\vec{e}_v$
   b) 1.12; $1.12\vec{e}_u$
4. −0.342
5. a) $2\vec{i}, -3\vec{j}$; 2, −3
   b) $\vec{i}, \vec{0}$; 1, 0
   c) $-15\vec{i}, 3\vec{j}$; −15, 3
6. a) $\vec{i}, -4\vec{j}, \vec{k}$; 1,−4,1
   b) $2\vec{i}, \vec{0}, 3\vec{k}$; 2,0,3
   c) $-2\vec{i}, -2\vec{j}, \vec{0}$; −2,−2,0
7. $\vec{u} = 4\vec{i} + 5\vec{j} + 0\vec{k}$
   $\vec{v} = -2\vec{i} - 3\vec{j} + \vec{k}$
8. a) a b) $b\vec{j}$ c) $-c\vec{k}$
9. a) $\frac{1}{\sqrt{2}}$ b) $\frac{1}{\sqrt{2}}$
11. a) 2
    b) $\frac{2}{\sqrt{29}}$
    c) $4\vec{j}$
    d) $\frac{4}{25}\overrightarrow{(-3,4)}$
12. $\vec{u} \perp \vec{v}$, or $|\vec{u}| = 0$
13. a) $|\vec{u}| = |\vec{v}|$ or $\vec{u} \perp \vec{v}$
    b) $\vec{u} = \vec{v}$ or $\vec{u} \perp \vec{v}$
14. 1 and 1
15. $\sqrt{\frac{41}{5}}$
16. a) 1
    b) $\overrightarrow{\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)}$
17. a) 13
    b) $-12\vec{j} + 2\vec{k}$

3.2 Exercises, page 124

1. a) 12
   b) 18.7
   c) 0
   d) 32
   e) −3.14
2. a) 6
   b) −0.5
   c) 3
3. a) 0
   b) 1
   c) −1
4. a) vector
   b) scalar
   c) vector
   d) vector
   e) scalar
   f) scalar
   g) vector
   h) vector
   i) vector
10. No: $\vec{u}$ or $\vec{v}$ could be $\vec{0}$.
11. a) 50
    b) −50
12. a) $0°$
    b) $100°$
    c) $90°$
13. b) e) f) are meaningful,
    a) c) d) are not.
14. a) 4; 4
    b) no

3.3 Exercises, page 128

1. a) 14
   b) −12
   c) 0
   d) 6
   e) 0
   perpendicular
2. 1
3. a) 0
   b) 1
4. $t = 2$
5. a) $40°$
   b) $90°$
   c) $105°$
   d) $82°$
6. a) 3
   b) $-\frac{2}{5}$
7. a) $k = -5$ or $k = 3$
   b) $\theta = 88°$ or $\theta = 85°$
8. a) 13
   b) 26
   c) $x^2 + y^2 + z^2$
9. a) $\vec{p} = \overrightarrow{(5,4)}$
   (or any multiple)
   b) $\vec{e}_p = \frac{1}{\sqrt{41}}\overrightarrow{(5,4)}$
10. a) −10
    b) 9
    c) 12
    d) −2
    e) 38
13. a) 7
    b) $\frac{1}{\sqrt{7}}(\vec{p} - 3\vec{q})$
15. $k = 2.566$ or 0.234

3.4 Exercises, page 131

1. a) $\alpha = 71°, \beta = 61°, \gamma = 144°$
   b) 2, 3, −5
2. a) $81°$
   b) $40°$
   c) $40°$
   d) $135°$
3. a) 1
   b) 6
   c) 6
   d) −4
4. a) $\frac{7}{\sqrt{5}}; \frac{7}{\sqrt{34}}$
   b) $-\frac{3}{\sqrt{45}}; -\sqrt{3}$
5. $\overrightarrow{BA} \cdot \overrightarrow{BC} < 0$
6. a) $\measuredangle P = 123°$,
   $\measuredangle Q = 29°$,
   $\measuredangle R = 28°$
   b) $\measuredangle P = 154°$,
   $\measuredangle Q = 16°$,
   $\measuredangle R = 10°$
7. $\vec{u} = 4\vec{i} + 5\vec{j} + 0\vec{k}$
   $\vec{v} = -2\vec{i} - 3\vec{j} + \vec{k}$
8. a) $\overrightarrow{\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)}$ b) $\overrightarrow{\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)}$
9. a) Either the vectors have the same magnitude, or they are perpendicular.
   b) Either the vectors are equal, or they are perpendicular.
10. a) −7.21
    b) $-2\overrightarrow{(-3,-2)} = \overrightarrow{(6,4)}$

3.4 Exercises, page 131, continued

11. $\overrightarrow{(-2,3)} = \frac{1}{2}\,\overrightarrow{(1,1)} + \frac{5}{2}\,\overrightarrow{(-1,1)}$

12. a) yes
 b) no
 c) yes

13. a) $\overrightarrow{OP} = -\vec{r}$
 b) $\overrightarrow{RC} = \vec{c} - \vec{r}$; $\overrightarrow{PC} = \vec{c} + \vec{r}$
 c) 0, since $|\vec{r}| = |\vec{c}|$
 d) $\overrightarrow{RC} \perp \overrightarrow{PC} \Rightarrow \measuredangle C$ a right angle

14. $\cos \measuredangle ABD = \dfrac{1}{\sqrt{10}} = \cos \measuredangle ACD$
 $\measuredangle B = \measuredangle C \Rightarrow A, B, C, D$ concyclic

3.5 Exercises, page 135

1. a) $5\vec{e}$; $\vec{u}, \vec{v}, \vec{e}$ RH system, with $\vec{e} \perp \vec{u}$ and $\vec{e} \perp \vec{v}$
 b) $42\vec{e}$; as in part a)
 c) $16.1\vec{e}$; as in part a)
 d) $17.3\vec{e}$; as in part a)
 e) $33\vec{i}$
 f) $-33\vec{i}$

2. a) $\vec{0}$
 b) $\vec{i}$
 c) $-\vec{k}$

3. 26.8

4. 74.8

5. b) c) e) are meaningful,
 a) d) f) are not.

11. b) no
 c) no

15. a) No, but $\vec{a}, \vec{b}, \vec{c}$ must be coplanar, with $|\vec{b}| \sin \theta = |\vec{c}| \sin \phi$.
 b) Yes, since $\theta = \phi$ and $\vec{a}, \vec{b}, \vec{e}$, in same RH system as $\vec{a}, \vec{c}, \vec{e}_2$.

16. Both sides equal $\vec{i} - \vec{j}$.

3.6 Exercises, page 145

1. a) $\vec{k}$
 b) $3\overrightarrow{(4,-1,-1)}$
 c) $\overrightarrow{(-46,-11,-14)}$
 d) $\vec{0}$

2. a) $\pm\vec{k}$
 b) $\pm\dfrac{1}{\sqrt{18}}\,\overrightarrow{(4,-1,-1)}$
 c) $\pm\dfrac{1}{\sqrt{2433}}\,\overrightarrow{(46,11,14)}$
 d) any vector $\dfrac{1}{\sqrt{a^2+b^2+c^2}}\,\overrightarrow{(a,b,c)}$, such that $a - 3b + 2c = 0$

3. a) $\vec{p}\times\vec{q}$
 b) $\vec{0}$
 c) 0
 d) $\vec{p}\times\vec{r}\cdot\vec{q}$

4. 64.8

5. a) 9.76
 b) 61.0

6. 2

7. $\dfrac{32}{3}$

8. a) $\alpha = 24°$
 b) $\beta = 17°$
 c) $\alpha = 28°$
 d) $\theta = 81°$

9. No; $\sin\theta = \dfrac{7}{10}$ gives two angles in [0°, 180°]

10. a) $(\vec{u}\times\vec{v})\times\vec{w} = \overrightarrow{(-28,-14,-14)}$
 $\vec{u}\times(\vec{v}\times\vec{w}) = \overrightarrow{(-30,-20,-4)}$
 b) not associative

11. $\vec{i}\times(\vec{u}\times\vec{k}) = \overrightarrow{(0,0,-1)} \neq \vec{0}$

13. $\vec{a}\times\vec{c} = -5(\vec{a}\times\vec{b})$, and if normals collinear then $\vec{a}, \vec{b}, \vec{c}$ coplanar

15. $4\overrightarrow{(3,3,1)}$

Inventory, page 147

1. tail
2. $2\vec{i}$
3. −3
4. resolving
5. scalar
6. vector
7. cross
8. $\vec{v}$ on $\vec{u}$
9. parallelogram; $\vec{u}$; $\vec{v}$
10. cross
11. 1
12. $\vec{0}$
13. 0
14. negative
15. triple scalar product
16. $\vec{a}\cdot\vec{b}\times\vec{c}$
17. perpendicular

Review Exercises, pages 148–151

1. a) $6\vec{i}, -5\vec{j}, -3\vec{k}$; 6, −5, −3
 b) $-\sqrt{2}\vec{i}, \vec{j}, \sqrt{3}\,\vec{k}$; $-\sqrt{2}$, 1, $\sqrt{3}$
 c) $-3\vec{i}, 3\vec{j}, -6\vec{k}$; −3, 3, −6

2. a) 1.147; 1.638
 b) −3,782; 1.302
 c) −5; 0

3. a) $\vec{u}, \vec{v}$ collinear, same direction
 b) $\vec{u}, \vec{v}$ collinear, opposite direction

4. a) 5.73
 b) −5
 c) −92.1

5. a) 5.73
 b) −1
 c) −9.21

6. a) 2
 b) 2
 c) 2
 d) 4
 e) 0
 f) 8

7. a) 12
 b) −39
 c) 0
 d) 0

8. $k = 1$

12. $\vec{e}_v = \dfrac{1}{\sqrt{5+2\sqrt{2}}}(2\vec{a} - \vec{b})$

13. a) 97°
 b) $-\dfrac{1}{\sqrt{13}}$
 c) $-\dfrac{1}{13}\,\overrightarrow{(-2,-3)}$
 d) $-\dfrac{1}{\sqrt{5}}$
 e) $-\dfrac{1}{5}\overrightarrow{(2,-1)}$

14. 2.24 or −3.57

15. c) 3

16. a) $\measuredangle P = 4°$
$\measuredangle Q = 29°$
$\measuredangle R = 147°$
b) $\measuredangle P = 90°$,
$\measuredangle Q = 35°$,
$\measuredangle R = 55°$

17. a) $\overrightarrow{AN} = \frac{1}{2}\vec{b} - \vec{a}$;
$\overrightarrow{BN} = \frac{1}{2}\vec{a} - \vec{b}$

18. $2|\vec{a}|^2 + 2|\vec{c}|^2$

19. a) $2.54\,\vec{e}$; $\vec{u}, \vec{v}, \vec{e}$ RH system with $\vec{e} \perp \vec{u}$ and $\vec{e} \perp \vec{v}$
b) $3.76\,\vec{e}$; as in part a)
c) $29.6\,\vec{e}$; as in part a)
d) $\overrightarrow{(4,12,-3)}$

20. $\vec{e}_1 = \frac{1}{\sqrt{2241}}\overrightarrow{(-14,37,-26)}$
$\vec{e}_2 = -\frac{1}{\sqrt{2241}}\overrightarrow{(-14,37,-26)}$

21. 54.2

22. no

23. a) yes
b) no
c) no
d) yes
e) yes

25. a) $\vec{p} = \overrightarrow{(5,-4,12)}$
(or any multiple)
b) $\vec{e}_p = \frac{1}{\sqrt{185}}\overrightarrow{(5,-4,12)}$

26. a) $\theta = 0°$
b) $\theta = 180°$

27. θ not defined

29. c) $\vec{v}$ must be perpendicular to $\vec{u}$ and to $\vec{w}$

30. b) $\vec{0} + \vec{p} \times \vec{q} + \vec{p} \times \vec{r} = \vec{0}$
c) $\vec{q} \times \vec{p} + \vec{0} + \vec{q} \times \vec{r} = \vec{0}$
$\vec{r} \times \vec{p} + \vec{r} \times \vec{q} + \vec{0} = \vec{0}$

33. a) $\gamma = 109.5°$
b) $OA = \sqrt{3}$, $OB = \sqrt{3}$, $AB = \sqrt{8}$
$\gamma = 109.5°$, $\alpha = \beta \doteq 35°$
c) each side $\sqrt{8}$,
each angle $60°$
d) $\overrightarrow{OD} = \vec{i}$
$\overrightarrow{OE} = \vec{j}$
$\overrightarrow{OF} = \vec{k}$

Volume $= \frac{1}{6}$ units^3

34. a) sketch not provided
b) $\begin{bmatrix} 4 \\ 8 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -2 \end{bmatrix} = 0$
c) $D(-4,-8)$
d) $\alpha = -\frac{1}{2}$
e) $E(0,10)$; $F(-4,2)$
f) Area $OAEF = 40$
Area $OBDC = 40$

35. ii) $AG : GL = 2 : 1$

36. i) $p = 4, q = 3$
$D(-3,-3)$
ii) $E\left(-\frac{17}{8}, \frac{25}{8}\right)$ $F\left(-\frac{21}{4}, \frac{25}{4}\right)$

37. a) 8
b) 5

Chapter Four
Applications of Vectors

4.1 Exercises, page 160

1.

	a)	b)	c)	d)	e)
$\vert\vec{R}\vert$	13	17	17.7	40.3	424
resultant bearing	067°	107°	316°	358°	179°
equilibrant bearing	247°	287°	136°	178°	359°

2. 73.7 N, 52° between P and R (or 23° between Q and R)
3. 295 N; 24° to horizontal (or 66° to vertical)
4. 90° between 10 and 24
   157° between 24 and 26
   113° between 26 and 10
5. 56° between 15 and 11
   147° between 11 and 23
   157° between 23 and 15
6. 57.2 N, bearing 061°
7. 70.7 N
8. 49.7 N
9. 63.0 N for 40° string
   75.1 N for 50° string
10. 78.7 N for 29° string
    91.2 N for 41° string
11. 30.0 N for 80 cm cord
    35.5 N for 70 cm cord
12. 5 N
13. bearing 125°

4.2 Exercises, pages 168–170

1. a) 10; bearing 037°
   b) 91.2; bearing 351°
3. a) 11.0
   b) $\alpha = 63°, \beta = 35°, \gamma = 69°$
4. a) $17.3\vec{e}_1 + 10\vec{e}_2$
   b) $98.5\vec{e}_1 + 17.4\vec{e}_2$
   c) $17.4\vec{e}_1 + 98.5\vec{e}_2$
   d) $-32.1\vec{e}_1 + 38.3\vec{e}_2$
5. 73.7 N, 52° between P and R (or 23° between Q and R)
6. 57.2 N, bearing 061°
7. 20.3, bearing 021°
8. 52.1 N, bearing 070°
9. $-8.94\vec{i} - 49.1\vec{j}$
10. a) $26.2\vec{i} - 236\vec{j} + 183\vec{k}$
    b) $\alpha = 85°, \beta = 142°, \gamma = 52°$
11. 252 000 N;
    $\alpha = 83°, \beta = 37°, \gamma = 53°$
12. a) $\overrightarrow{(18,0,-6)}$
    b) $\overrightarrow{(-3,-4,2)}$
13. a) $180° - \theta$
    b) $|\vec{r}||\vec{F}| \sin\theta\, \vec{e}$, where $\vec{r}, \vec{F}, \vec{e}$ form a RH system, and $|\vec{e}| = 1$
    c) maximum at $\theta = 90°$; minumum at $\theta = 0°$ or 180°
14. $7.07\vec{e}_1 + 7.07\vec{e}_2$
15. $4.30\vec{e}_1 + 6.45\vec{e}_2$;
    $\theta = 34°$
16. 843 N
17. bearing 125°
18. 52 N
19. a) $|\vec{N}| = 176$ N
       $|\vec{F}| = 85.9$ N
    b) 0.488
    c) 77.2 N
20. a) $|\vec{N}| = mg\cos\theta$
       $|\vec{F}| = mg\sin\theta$
    b) $\mu = \tan\theta$
    c) $mg\sin\theta\cos\theta$
       (or $\frac{1}{2}mg\sin 2\theta$)
21. a) 56.6 N (horizontal); 113 N
    b) 117 N (horizontal); 152 N

4.3 Exercises, page 174

1. a) 140
   b) 15
   c) 294
2. a) $743\vec{e}_1 + 669\vec{e}_2$
   b) 22 300 J
3. 1 360 000 J
4. 507 J
5. 0; situation 'impossible', since force acts perpendicularly on intended direction of motion.
6. 421
7. a) 384 J
   b) 697 J
8. a) 2.5 m
   b) 6.26 m/s
9. a) 1.47 J
   b) 5.42 m/s

4.4 Exercises, page181

1. a) 47 km/h, bearing 058°
   b) 106 km/h, bearing 261°
   c) 888 km/h, bearing 093°
   d) 13 km/h, bearing 030°
2. a) 105 km/h, westward
   b) 85 km/h, westward
   c) 101 km/h, bearing 263°
3. 206 km/h, bearing 104°
4. 4 h 10 min
5. a) bearing 239°
   b) 28 min
6. a) at an angle of 70° to *AB*, upstream
   b) 5 min
7. a) at an angle of $\theta°$ to *AB*, upstream
   b) $\dfrac{w}{v \sin 2\theta}$ min
8. a) 50 km/h, bearing 287°
   b) 50 km/h, bearing 107°
9. 23 km/h

4.5 Exercises, pages 186–187

1. a) $\overrightarrow{(1,6)}$; 6.08
   b) $\overrightarrow{(-4,-5,7)}$; 9.49
   c) $\overrightarrow{(-12,4,4)}$; 13.3
2. a) 340 km/h, bearing 115°
   b) 340 km/h, bearing 295°
3. $\overrightarrow{(43,45,-2)}$; 62.3 km/h
4. a) $\overrightarrow{(-21,47,0)}$; 51.5
   b) $\overrightarrow{(-22,48,-2)}$; 52.8
   c) $\overrightarrow{(-20,46,-1)}$; 50.2
5. 206 km/h, bearing 104°
6. a) bearing 239°
   b) 28 min
7. a) $\vec{0}$
   b) $-20\vec{i}$
   c) $-9.33\vec{j}$
   d) $-20\vec{i} - 9.33\vec{j}$; 22.1 m/s
8. a) 50 km/h, bearing 287°
   b) 50 km/h, bearing 107°
9. 15.017 km/h, bearing 070°
10. a) 547 km/h, bearing 106°
    b) 547 km/h, bearing 106°
    (Wind makes no difference, here)
11. 613 km/h, bearing 072°
12. 53.3 km/h, wind from bearing 214°

In Search of, page 191

2. $\dfrac{v^2 \sin^2 \alpha}{2g}$
3. $\dfrac{2v^2 \cos \alpha \sin \alpha}{g} \left(\text{or } \dfrac{v^2 \sin 2\alpha}{g}\right)$
4. a) describes a circle
   b) $\overrightarrow{v(t)} = -a \sin t\vec{i} + a \cos t\vec{j}$
   perpendicular to $\overrightarrow{r(t)}$, with same magnitude a
   c) $-\overrightarrow{r(t)}$
   centripetal
5. a) describes an ellipse
   b) $\overrightarrow{v(t)} = -a \sin t\vec{i} + b \cos t\vec{j}$
   c) $-\overrightarrow{r(t)}$
   centripetal

Inventory, pages 192–193

1. vector
2. particle
3. newtons
4. 9.8 N
5. added
6. resultant
7. equilibrant
8. equilibrium
9. space (or position)
10. $\vec{v} = \cos \alpha\vec{i} + \sin \alpha\vec{j}$
11. $\vec{v} = \cos \alpha\vec{i} + \cos \beta\vec{j} + \cos \gamma\,\vec{k}$
12. components
13. parallel
14. direction; $\vec{i}, \vec{j}, \vec{k}$
15. magnitude; displacement
16. scalar
17. joules
18. dot product
19. zero
20. relative
21. $\vec{v}_{BC}$; $\vec{v}_{AB}$; $\vec{v}_{BC}$

Review Exercises, pages 194–195

1. 112 N, at an angle of 23° to $\vec{p}$
2. a) 739 N, at an angle of 51° to horizontal
   b) 51° *below* horizontal; 739 N
3. 158° between 85 and 40
   39° between 40 and 50
   163° between 50 and 85
4. No; no possible triangle
5. a) $|\vec{P}| = 83.6$ N; $|\vec{Q}| = 33.4$ N;
   b) $|\vec{P}| = 24.6$ N; $|\vec{Q}| = 49.2$ N;
6. 161°
7. $|\vec{T}_1| = 47.3$ N
   $|\vec{T}_2| = 65.7$ N
8. a) 81.3 N
   b) $\alpha = 36°, \beta = 116°, \gamma = 112°$
9. $|\vec{J}| \doteq 2253$, bearing 340°
10. $\vec{F} \doteq 22.8\vec{i} - 76\vec{j} + 60.8\vec{k}$
11. $\vec{F} = \dfrac{100}{\sqrt{3}}\vec{e}_1 + \dfrac{100}{\sqrt{3}}\vec{e}_2 + \dfrac{100}{\sqrt{3}}\vec{e}_3$
12. 526 N, up slope
13. $\vec{N} \doteq 416$ N
    $\vec{F} \doteq 260$ N
14. a) 328 N for 45° string
    268 N for 60° string
    b) tension 80.6 N
    thrust 63.2 N
16. a) 1270 J
    b) 495 J
17. a) $\vec{F} \doteq 111\vec{e}_1 + 792\vec{e}_2$
    b) 1110 J
18. 1830 J
19. 98.5 km/h, bearing 294°
20. a) 088°
    b) 1 hr 56 min
21. a) 55.7 km/h, bearing 351°
    b) 55.7 km/h, bearing 171°
22. $\vec{v}_{FS} = 24\vec{i} - \dfrac{95}{2}\vec{j} - \vec{k}$
    $|\vec{v}_{FS}| = 53.2$ km/h

Chapter Five Equations of Lines in 2- and 3-Space

5.1 Exercises, page 202

1. a) $(2,-1)$; $\overrightarrow{(4,2)}$
   b) $(8,-3)$; $\overrightarrow{(5,-4)}$
   c) $(3,-1,4)$; $\overrightarrow{(5,-2,1)}$
   d) $(-4,7,5)$; $\overrightarrow{(1,0,-8)}$
   (Answers will vary.)
2. a) $\vec{r} = \overrightarrow{(3,7)} + k\overrightarrow{(1,5)}$
   b) $\vec{r} = \overrightarrow{(-2,0)} + k\overrightarrow{(-9,-2)}$
   c) $\vec{r} = \overrightarrow{(6,9)} + k\overrightarrow{(-2,4)}$
   d) $\vec{r} = \overrightarrow{(3,2,7)} + k\overrightarrow{(1,5,3)}$
   e) $\vec{r} = \overrightarrow{(0,-2,0)} + k\overrightarrow{(-9,-2,5)}$
   f) $\vec{r} = \overrightarrow{(2,4,-3)} + k\overrightarrow{(0,0,6)}$
3. a) $\vec{r} = \overrightarrow{(4,-6)} + k\overrightarrow{(-2,13)}$
   b) $\vec{r} = \overrightarrow{(4,-6,2)} + k\overrightarrow{(-5,8,5)}$
4. a) $\vec{r} = \overrightarrow{(-5,2)} + k\overrightarrow{(2,3)}$
   b) $\vec{r} = \overrightarrow{(3,-5,2)} + k\overrightarrow{(2,3,6)}$
5. a) $\vec{r} = \overrightarrow{(3,-1)} + k\overrightarrow{(-3,2)}$
   b) $\vec{r} = \overrightarrow{(3,-1,-5)} + k\overrightarrow{(0,-3,2)}$
6. $\vec{r} = \overrightarrow{(3,-1)} + k\overrightarrow{(2,3)}$
7. $\vec{r} = \overrightarrow{(3,0,2)} + k\overrightarrow{(3,14,1)}$
8. $\vec{r} = \overrightarrow{(3,-1,2)} + k\overrightarrow{(11,19,1)}$
9. a) 12
   b) $\frac{13}{3}$
10. b) between P_1 and P_2
    c) $k > 1 \Rightarrow$ outside segment P_1P_2, closer to P_2
    $k < 0 \Rightarrow$ outside segment P_1P_2, closer to P_1
11. a) $\vec{r} = \overrightarrow{(3,8)} + k\overrightarrow{(1,0)}$
    b) $\vec{r} = \overrightarrow{(3,8,1)} + k\overrightarrow{(0,1,0)}$
    c) $\vec{r} = \overrightarrow{(3,8,1)} + k\overrightarrow{(0,0,1)}$

5.2 Exercises, pages 206–207

1. a) $(5,2)$ $\overrightarrow{(2,4)}$
   b) $(-3,1)$ $\overrightarrow{(8,-5)}$
   c) $(-2,0)$ $\overrightarrow{(1,3)}$
   d) $(-4,1)$ $\overrightarrow{(6,0)}$
   e) $(5,2,2)$ $\overrightarrow{(2,4,-5)}$
   f) $(-3,1,-2)$ $\overrightarrow{(8,-5,5)}$
   g) $(-2,0,4)$ $\overrightarrow{(1,3,2)}$
   h) $(-4,1,0)$ $\overrightarrow{(6,0,-7)}$
2. a) 2, 4
   b) 8, −5
   c) 1, 3
   d) 6, 0
   e) 2, 4, −5
   f) 8, −5, 5
   g) 1, 3, 2
   h) 6, 0, −7
3. a) $(-6,3)$ $(-2,1)$ $(2,-1)$
   b) $\begin{cases} x = -2 + 4k \\ y = 1 - 2k \end{cases}$
   $\begin{cases} x = 2 + 4s \\ y = -1 - 2s \end{cases}$
   $\begin{cases} x = -6 + 8t \\ y = 3 - 4t \end{cases}$
4. a) $(-6,3,5)$ $(-2,1,4)$ $(-10,5,6)$
   b) $\begin{cases} x = -6 + 8k \\ y = 3 - 4k \\ z = 5 - 2k \end{cases}$
   $\begin{cases} x = -2 + 4s \\ y = 1 - 2s \\ z = 4 - s \end{cases}$
   $\begin{cases} x = -10 + 4t \\ y = 5 - 2t \\ z = 6 - t \end{cases}$
5. a) $\vec{r} = \overrightarrow{(-3,4)} + k\overrightarrow{(5,1)}$
   $\begin{cases} x = -3 + 5k \\ y = 4 + k \end{cases}$
   b) $\vec{r} = \overrightarrow{(-3,4)} + t\overrightarrow{(10,-2)}$
   $\begin{cases} x = -3 + 10t \\ y = 4 - 2t \end{cases}$
   c) $\vec{r} = \overrightarrow{(-3,4)} + s\overrightarrow{(6,-2)}$
   $\begin{cases} x = -3 + 6s \\ y = 4 - 2s \end{cases}$
   d) $\vec{r} = \overrightarrow{(7,2)} + a\overrightarrow{(4,-5)}$
   $\begin{cases} x = 7 + 4a \\ y = 2 - 5a \end{cases}$
   e) $\vec{r} = \overrightarrow{(8,-3)} + m\overrightarrow{(1,0)}$
   $\begin{cases} x = 8 + m \\ y = -3 \end{cases}$
   f) $\vec{r} = \overrightarrow{(8,-3)} + k\overrightarrow{(0,1)}$
   $\begin{cases} x = 8 \\ y = -3 + k \end{cases}$
6. a) $\vec{r} = \overrightarrow{(5,-3,4)} + k\overrightarrow{(-6,2,1)}$
   $\begin{cases} x = 5 - 6k \\ y = -3 + 2k \\ z = 4 + k \end{cases}$
   b) $\vec{r} = \overrightarrow{(5,-3,4)} + t\overrightarrow{(2,5,-5)}$
   $\begin{cases} x = 5 + 2t \\ y = -3 + 5t \\ z = 4 - 5t \end{cases}$
   c) $\vec{r} = \overrightarrow{(5,-3,4)} + s\overrightarrow{(6,7,-2)}$
   $\begin{cases} x = 5 + 6s \\ y = -3 + 7s \\ z = 4 - 2s \end{cases}$
   d) $\vec{r} = \overrightarrow{(7,2,-1)} + m\overrightarrow{(4,-5,1)}$
   $\begin{cases} x = 7 + 4m \\ y = 2 - 5m \\ z = -1 + m \end{cases}$
   e) $\vec{r} = \overrightarrow{(8,-3,4)} + k\overrightarrow{(1,0,0)}$
   $\begin{cases} x = 8 + k \\ y = -3 \\ z = 4 \end{cases}$
   f) $\vec{r} = \overrightarrow{(8,-3,4)} + s\overrightarrow{(0,1,0)}$
   $\begin{cases} x = 8 \\ y = -3 + s \\ z = 4 \end{cases}$
   g) $\vec{r} = \overrightarrow{(8,-3,4)} + t\overrightarrow{(0,0,1)}$
   $\begin{cases} x = 8 \\ y = -3 \\ z = 4 + t \end{cases}$
7. yes: *A, D*
   no: *B, C, E*
8. yes: *A, C, D*
   no: *B, E*
9. a) no
   b) yes
   c) no
13. a) $\begin{cases} x = k \\ y = 2.5 - 1.5k \end{cases}$
    b) $\begin{cases} x = 3 + 2t \\ y = -2 - 3t \end{cases}$
14. a) $\begin{cases} x = -3 - 4k \\ y = 2 + 7k \end{cases}$
    b) $k = \frac{x + 3}{-4}$
    $k = \frac{y - 2}{7}$
    c) $\frac{x + 3}{-4} = \frac{y - 2}{7}$
15. a) $\begin{cases} x = -3 + 2k \\ y = 2 - 4k \\ z = 1 + 7k \end{cases}$
    b) $k = \frac{x + 3}{2}$
    $k = \frac{y - 2}{-4}$
    $k = \frac{z - 1}{7}$
    c) $\frac{x + 3}{2} = \frac{y - 2}{-4} = \frac{z - 1}{7}$
16. c) $\left(\frac{41}{9}, \frac{30}{9}\right)$

5.3 Exercises, pages 213–214

1. a) (3,2); 5, 6
   b) (−1,4); 2, −7
   c) (2,0); −8, 3
   d) (3,3); 2.5, −4
   e) (2,4,1,); 3, 8, 5
   f) (−3,2,1); −3, 3.5, −9
   g) (2,4,7); 3, 5, 0
   h) (1,2,−2); −3, 0, 4
2. a) $\vec{r} = \overrightarrow{(3,2)} + k\overrightarrow{(5,6)}$
   b) $\vec{r} = \overrightarrow{(-1,4)} + k\overrightarrow{(2,-7)}$
   c) $\vec{r} = \overrightarrow{(2,0)} + k\overrightarrow{(-8,3)}$
   d) $\vec{r} = \overrightarrow{(3,3)} + k\overrightarrow{(2.5,-4)}$
   e) $\vec{r} = \overrightarrow{(2,4,1)} + k\overrightarrow{(3,8,5)}$
   f) $\vec{r} = \overrightarrow{(-3,2,1)} + k\overrightarrow{(-3,3.5,-9)}$
   g) $\vec{r} = \overrightarrow{(2,4,7)} + k\overrightarrow{(3,5,0)}$
   h) $\vec{r} = \overrightarrow{(1,2,-2)} + k\overrightarrow{(-3,0,4)}$
3. a) $\frac{x-3}{-2} = \frac{y-4}{5}$
   b) $\frac{x-6}{4} = \frac{y+3}{-1}$
   c) $\frac{x-2}{4} = \frac{y-1}{2} = \frac{z}{3}$
   d) $\frac{x+3}{-1} = \frac{y-4}{5}, z = 2$
   e) none exists
   f) $\frac{x}{-4} = \frac{y+1}{3}$
4. a) $\frac{x-5}{-6} = \frac{y+3}{2} = \frac{z-4}{1}$
   b) $\frac{x-4}{1} = \frac{z}{-3}, y = -1$
   c) $\frac{x-5}{2} = \frac{y+3}{5} = \frac{z-4}{-5}$
   d) $\frac{x-7}{4} = \frac{y-2}{-5} = \frac{z+1}{1}$
   e) f) g) none exist
5. $\vec{r} = \overrightarrow{(1,-3,5)} + k\overrightarrow{(4,2,-3)}$
6. 2
7. −0.5
9. 3
10. a) 3, 4
    b) 5, −2
    c) 1, −5
    d) 2, 0
    e) 0, 1
11. a) $-4x + y + 13 = 0$
    b) $6x + 4y + 20 = 0$
    c) $2x - 3y + 29 = 0$
    d) $-2x - 4 = 0$
12. parallel: a) d) e)
    perpendicular: b) c) f)
13. b) $\vec{r} = \overrightarrow{(3,5)} + k\overrightarrow{(5,-2)}$
14. a) $\vec{r} = \overrightarrow{(1,-3)} + t\overrightarrow{(7,4)}$
    b) $\vec{r} = \overrightarrow{(1,-3)} + k\overrightarrow{(4,-7)}$
15. 7
16. −7.5
17. $6x - 5y - 8 = 0$
18. $\frac{x-1}{7} = \frac{y-1}{-4}$
19. b) no

5.4 Exercises, page 217

1. a) −3, −2, 4; 6, 4, −8; 1.5, 1, −2
   (Answers may vary.)
   b) 0.5571, 0.3714, −0.7428
   c) 56°, 68°, 138°
   (or 124°, 112°, 42°)
2. a) −0.1690, 0.5071, 0.8452
   100°, 60°, 32°
   b) 0, −0.5300, 0.8480;
   90°, 122°, 32°
   c) 0.9370, −0.3123, −0.1562;
   20°, 108°, 99°
   d) −0.4472, 0, 0.8944;
   117°, 90°, 27°
3. a) $\frac{5}{\sqrt{30}}, -\frac{2}{\sqrt{30}}, \frac{1}{\sqrt{30}}$
   b) $\frac{1}{\sqrt{65}}, 0, -\frac{8}{\sqrt{65}}$
   c) $\frac{2}{\sqrt{45}}, \frac{4}{\sqrt{45}}, -\frac{5}{\sqrt{45}}$
   d) $\frac{8}{\sqrt{114}}, -\frac{5}{\sqrt{114}}, \frac{5}{\sqrt{114}}$
   e) $\frac{3}{\sqrt{98}}, \frac{8}{\sqrt{98}}, \frac{5}{\sqrt{98}}$
   f) $\frac{-3}{\sqrt{102.25}}, \frac{3.5}{\sqrt{102.25}}, \frac{-9}{\sqrt{102.25}}$
   g) $-\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}}$
4. ± 0.8660
5. 69° (or 111°)
6. $\vec{r} = k\overrightarrow{(\pm 0.6855,0.7071,0.1736)}$
7. 60°
8. 63°
9. $-\frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}, -\frac{2}{\sqrt{14}}$;
   $\frac{4}{\sqrt{525}}, \frac{22}{\sqrt{525}}, \frac{5}{\sqrt{525}}$
11. $-\frac{12}{\sqrt{229}}, -\frac{6}{\sqrt{229}}, \frac{7}{\sqrt{229}}$
13. $\overrightarrow{\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)}$

5.5 Exercises, page 221

1. a) Lines do not intersect:
   no solutions.
   b) Lines are identical:
   an infinity of solutions.
   c) Lines intersect in a point:
   one solution.
2. a) intersect at (3,−2)
   b) parallel and distinct
   c) parallel and identical
   d) intersect at (1.2,1.4)
3. a) consistent and independent
   b) inconsistent
   c) consistent and dependent
   d) consistent and independent
4. a) (−1,5)
   b) (4,−2)
   c) parallel and distinct
   d) (3,−1)
5. (−2,−1)
6. (5,15)
7. (−1,2)
8. a) $\vec{r} = \overrightarrow{(1,4)} + k\overrightarrow{(2,2)}$
   b) $\vec{r} = \overrightarrow{(-5,6)} + t\overrightarrow{(7,-5)}$
   c) $\vec{r} = \overrightarrow{(3,-2)} + s\overrightarrow{(5,-7)}$
   d) $\left(-\frac{1}{3}, \frac{8}{3}\right)$
9. a) $\vec{r} = k\overrightarrow{(8,4)}$;
   $\vec{r} = \overrightarrow{(5,0)} + t\overrightarrow{(-2,4)}$
   b) (4,2)
10. a) $\vec{r} = \overrightarrow{(3,10)} + t\overrightarrow{(4,3)}$
    b) $\left(\frac{127}{25}, \frac{289}{25}\right)$
    c) $\frac{13}{5}$
11. $\vec{r} = k\overrightarrow{\left(\frac{7}{8}, \frac{9}{4}\right)}$

5.6 Exercises, pages 227–228

1. a) (−1,5,4)
   b) (5,−2,4)
   c) skew
   d) parallel
   e) (3,−1,−4)
   f) skew

5.6 Exercises, pages 227–228, continued

2. consistent and independent: a) b) e)
   inconsistent: c) d) f)
3. (2,−1,5)
4. (5,2,15)
5. (3,−1,2)
6. $\frac{4}{\sqrt{1466}}$; $\frac{126}{\sqrt{405}}$
7. b) $\frac{39}{\sqrt{390}}$
8. b) $\sqrt{44}$
   c) $P_1(2,3,-2)$; $P_2(4,-3,-4)$
9. a) (4,−1,3)
   b) $\cos^{-1}\frac{25}{\sqrt{1470}} = 49°$
10. The lines intersect at (−2,4,−1).
11. a) L_1 at (6,11,1)
    b) $\frac{68}{\sqrt{326}}$
    c) (2−1,3), (4,1,−3)
12. a) $\vec{r} = k\left(\frac{8}{7}, 1, \frac{-4}{7}\right)$
    b) $\left(\frac{8}{7}, 1, \frac{-4}{7}\right)$
13. $\frac{1}{\sqrt{2}}d$
15. a) yes b) no c) no
16. a) $\overrightarrow{(5,-3,-1)}$
    d) For L_1, let $a = 0$ and $b = -\frac{1}{3}$.
    For L_2, let $p = 1$ and $q = -\frac{1}{3}$.
    (Answers may vary.)

5.7 Exercises, page 233

See answers to 2.6 Exercises

Inventory, pages 237–238

1. position vector of any point on the line;
   position vector of a given point on the line;
   a direction vector of the line;
   a parameter
2. (1,4); $\overrightarrow{(3,5)}$; k
3. (5,−1,0); $\overrightarrow{(-2,3,-6)}$; t
4. (−4,2); $\overrightarrow{(3,-5)}$; s; $-\frac{5}{3}$
5. (0,−1,4); (−6,1,0); t
6. (3,−2); $\overrightarrow{(4,-3)}$
7. (−1,0,3); $\overrightarrow{(4,4,1)}$
8. $-\frac{4}{3}$; $\overrightarrow{(4,3)}$
9. 1;
   the angle between the line and the x-axis direction;
   the angle between the line and the y-axis direction;
   the angle between the line and the z-axis direction
10. 1
11. parallel
12. parallel (or) skew
13. an infinite number of;
    an infinity of
14. one; one
15. no; no
16. the shortest distance between two skew lines L_1 and L_2;
    a point on L_1, a point on L_2;
    $\vec{m}_1 \times \vec{m}_2$
17. $\overrightarrow{(2,4,7)}$

Review Exercises, pages 239–243

1. a) $\vec{r} = \overrightarrow{(2,-5)} + k\overrightarrow{(3,2)}$
   $\begin{cases} x = 2 + 3k \\ y = -5 + 2k \end{cases}$
   b) $\vec{r} = \overrightarrow{(-5,-4)} + t\overrightarrow{(6,-2)}$
   $\begin{cases} x = -5 + 6t \\ y = -4 - 2t \end{cases}$
   c) $\vec{r} = \overrightarrow{(-7,0)} + s\overrightarrow{(2,-5)}$
   $\begin{cases} x = -7 + 2s \\ y = -5s \end{cases}$
   d) $\vec{r} = \overrightarrow{(2,1)} + k\overrightarrow{(-3,-2)}$
   $\begin{cases} x = 2 - 3k \\ y = 1 - 2k \end{cases}$
   e) $\vec{r} = \overrightarrow{(-4,1)} + k\overrightarrow{(1,0)}$
   $\begin{cases} x = -4 + k \\ y = 1 \end{cases}$
   f) $\vec{r} = \overrightarrow{(-4,1)} + t\overrightarrow{(0,1)}$
   $\begin{cases} x = -4 \\ y = 1 + t \end{cases}$
2. yes: A, B
   no: C, D
3. a) $\frac{x-1}{-2} = \frac{y-3}{-4}$
   b) $\frac{x-3}{-2} = \frac{y-1}{1}$
   c) $\frac{x+2}{1} = \frac{y+4}{-1}$
4. a) $4x + y + 9 = 0$
   b) $8x - 2y - 32 = 0$
   c) $4y + 20 = 0$
5. a) perpendicular
   b) parallel
   c) perpendicular
6. $\vec{r} = \overrightarrow{(-3,6,1)} + k\vec{c}$, where $\vec{c}$ is any linear combination of $\vec{a}$ and $\vec{b}$
7. b) $\vec{r} = k(\vec{b} + \vec{d})$
   c) $\vec{r} = \vec{b} + t(\vec{d} - \vec{b})$
8. c) $\vec{r} = \overrightarrow{(3,4)} + k\overrightarrow{(7,2)}$
9. a) $\frac{x+4}{-3} = \frac{y-3}{-1}$
   b) $\frac{x+4}{1} = \frac{y-3}{-3}$
10. $-\frac{6}{5}$
11. −2
12. a) $\vec{r} = \overrightarrow{(2,3,0)} + k\overrightarrow{(4,2,-2)}$
    $x = 2 + 4k$
    $y = 3 + 2k$
    $z = -2k$
    $\frac{x-2}{4} = \frac{y-3}{2} = \frac{z}{-2}$
    b) $\vec{r} = \overrightarrow{(-2,3,5)} + t\overrightarrow{(5,-4,1)}$
    $x = -2 + 5t$
    $y = 3 - 4t$
    $z = 5 + t$
    $\frac{x+2}{5} = \frac{y-3}{-4} = \frac{z-5}{1}$
    c) $\vec{r} = \overrightarrow{(1,0,10)} + s\overrightarrow{(1,2,-4)}$
    $x = 1 + s$
    $y = 2s$
    $z = 10 - 4s$
    $\frac{x-1}{1} = \frac{y}{2} = \frac{z-10}{-4}$
    d) $\vec{r} = \overrightarrow{(5,6,10)} + m\overrightarrow{(-2,1,3)}$
    $x = 5 - 2m$
    $y = 6 + m$
    $z = 10 + 3m$
    $\frac{x-5}{-2} = \frac{y-6}{1} = \frac{z-10}{3}$
    e) $\vec{r} = \overrightarrow{(-4,3,6)} + k\overrightarrow{(1,0,0)}$
    $x = -4 + k$
    $y = 3$
    $z = 6$
    none exists
    f) $\vec{r} = \overrightarrow{(-4,3,6)} + t\overrightarrow{(0,0,1)}$
    $x = -4$
    $y = 3$
    $z = 6 + t$
    none exists

13. a) yes
b) no
c) yes

14. a) $\vec{r} = \overrightarrow{(3,-2,1)} + k\overrightarrow{(1,2,6)}$

15. $\vec{r} = \overrightarrow{(-1,-2,4)} + s\overrightarrow{(-3,1,4)}$

16. $\vec{r} = \overrightarrow{(4,1,5)} + k\overrightarrow{(-25,14,2)}$

17. $\vec{r} = \overrightarrow{(3,-1,3)} + k\overrightarrow{(4,5,-1)}$

18. $\dfrac{3}{17}$

19. -2

20. 3

21. a), d)

22. a) $\vec{r} = \overrightarrow{(1,-2,3)} + k\overrightarrow{(1,3,2)}$
b) $\vec{r} = \overrightarrow{(1,-2,3)} + k\overrightarrow{(1,3,-2)}$

24. $\dfrac{k-2m}{\sqrt{14k^2+2km+5m^2}}$
$\dfrac{2k}{\sqrt{14k^2+2km+5m^2}}$
$\dfrac{3k+m}{\sqrt{14k^2+2km+5m^2}}$
$k, m \in \mathbb{R}$ with k and m not both zero

25. a) $\dfrac{2}{\sqrt{29}}, -\dfrac{3}{\sqrt{29}}, \dfrac{4}{\sqrt{29}}$
b) $-\dfrac{2}{\sqrt{30}}, \dfrac{5}{\sqrt{30}}, \dfrac{1}{\sqrt{30}}$
c) $\dfrac{2}{\sqrt{14}}, -\dfrac{1}{\sqrt{14}}, \dfrac{3}{\sqrt{14}}$

26. 57°

27. $\vec{r} = k\overrightarrow{(0.9063,0.0839,0.4135)}$

28. a) intersect at (2,1)
b) parallel and distinct
c) parallel and identical

29. a) consistent and independent
b) inconsistent
c) consistent and dependent

30. a) intersect at (6,−1)
b) intersect at (17,5)
c) parallel and distinct

31. a) (−2,14)
b) 27°

32. $\vec{r} = \vec{d} + k(\vec{d} - \vec{b})$

33. a) $S(0,4,0)$, $T(0,0,3)$, $U(2,0,3)$
b) $A(0,0,1.5)$, $B(2,4,1.5)$

34. a) intersect at (2,9,1)
b) skew
c) the same line
d) skew

35. a) consistent and independent
b) inconsistent
c) consistent and dependent
d) inconsistent

36. (4,6,2)

37. (3,−4,3)

38. b) $\dfrac{35}{\sqrt{83}}$

39. a) (1,7,−5)
b) 38°

40. intersect at (−7,−2,0); direction vectors are linearly dependent

41. intersect at (8,3,5); direction vectors are linearly independent

42. a) 7
b) $\vec{r} = \overrightarrow{(1,1,-3)} + k\overrightarrow{(2,3,6)}$

43. a) $x = \dfrac{y-2}{\frac{1}{3}} = \dfrac{z+2}{\frac{1}{2}}$
b) $x = \dfrac{y}{2} = \dfrac{-z+5}{4}$

44. 11 : 4

45. 5 : 6

46. 3 : 2

47. 3 : 2; 2 : 1

48. 7 : 6

49. $Q_1(3,5,7)$
$Q_2(-1,-3,-1)$

50. a) P_1 (3,0,2)
P_2 (1,−3,−4)

51. (i) (1,1,−1);
$2x - 2y + z = 1$
(ii) $\dfrac{1}{3}$

52. $\vec{r} = \overrightarrow{(-1,2,1)}$

53. E

54. a) $2y - x = 18$
b) $3y - 4x = -3$
c) $H(12,15)$
e) The three altitudes are concurrent at H.
f) $S(1,2)$

Chapter Six
Equations of Planes

6.1 Exercises, pages 249–250

1. plane: a) b) d)
   line: c)
2. a) $\overrightarrow{(3,4,5)}, \overrightarrow{(0,-4,6)}\overrightarrow{(2,1,5)}$
   b) $\overrightarrow{(0,1,9)}, \overrightarrow{(2,3,-4)}\overrightarrow{(-1,-2,9)}$
   c) $\overrightarrow{(4,-2,0)}, \overrightarrow{(3,4,-1)}\overrightarrow{(2,0,5)}$
3. a) $\vec{r} =$
   $\overrightarrow{(3,2,7)} + k\overrightarrow{(4,6,2)} + s\overrightarrow{(0,-2,5)}$
   $\begin{cases} x = 3 + 4k \\ y = 2 + 6k - 2s \\ z = 7 + 2k + 5s \end{cases}$
   b) $\vec{r} =$
   $\overrightarrow{(2,-3,1)} + k\overrightarrow{(5,-3,-1)} + s\overrightarrow{(2,3,1)}$
   $\begin{cases} x = 2 + 5k + 2s \\ y = -3 - 3k + 3s \\ z = 1 - k + s \end{cases}$
   c) $\vec{r} =$
   $\overrightarrow{(3,2,-7)} + k\overrightarrow{(1,5,-8)} + s\overrightarrow{(5,-3,-1)}$
   $\begin{cases} x = 3 + k + 5s \\ y = 2 + 5k - 3s \\ z = -7 - 8k - s \end{cases}$
4. a) $(5,-9,17)$
5. $\vec{r} =$
   $\overrightarrow{(4,0,5)} + k\overrightarrow{(-1,2,4)} + s\overrightarrow{(3,2,-6)}$
6. $\vec{r} =$
   $\overrightarrow{(3,2,1)} + k\overrightarrow{(2,-3,0)} + s\overrightarrow{(0,5,-2)}$
7. $\vec{r} = \overrightarrow{(5,6,7)} + t\overrightarrow{(1,0,5)} + k\overrightarrow{(2,0,-1)}$
8. $\vec{r} =$
   $\overrightarrow{(2,-3,1)} + k\overrightarrow{(3,4,-8)} + s\overrightarrow{(5,-2,4)}$
10. a) $\vec{r} =$
    $\overrightarrow{(2,5,1)} + k\overrightarrow{(3,-6,2)} + s\overrightarrow{(4,-2,1)}$
    b) $\vec{r} =$
    $\overrightarrow{(1,2,-3)} + k\overrightarrow{(3,-6,2)} + s\overrightarrow{(4,-2,1)}$
11. $\vec{r} = \overrightarrow{(3,0,0)} + k\overrightarrow{(3,2,0)} + s\overrightarrow{(3,0,-7)}$
12. $\vec{r} = \overrightarrow{(2,3,1)} + k\overrightarrow{(3,-7,1)} + s\overrightarrow{(0,0,1)}$
13. a) $\overrightarrow{CD} \parallel \vec{a}$
    b) $\vec{r} = \overrightarrow{(3,5,8)} + k\overrightarrow{(-2,2,5)} + s\vec{a}$,
    where $\vec{a} \nparallel \overrightarrow{(-2,2,5)}$
14. $\vec{r} = \overrightarrow{(2,3,1)} + k\overrightarrow{(6,3,0)} + s\overrightarrow{(0,4,6)}$
15. $\vec{r} =$
    $\overrightarrow{(1,3,2)} + k\overrightarrow{(2,1,-2)} + s\overrightarrow{(2,-5,10)}$
16. none
17. All sets are coplanar.

6.2 Exercises, pages 256–257

1. a) $\overrightarrow{(3,-5,4)}$
   b) line in plane
   c) $\overrightarrow{(1,-5,4)}$
   d) $\overrightarrow{(1,-2,-3)}$
   e) $\overrightarrow{(2,0,-6)}$
   f) $\overrightarrow{(0,2,5)}$
   g) $\overrightarrow{(-6,10,-8)}$
   h) $\overrightarrow{(2,-4,-6)}$
2. parallel and distinct: a) g)
   parallel and identical: d) h)
4. $3x + 6y + z - 21 = 0$
5. $4x - 2y + 7z = -46$
6. a) $A + 4B - 6C + D = 0$
   b) $3A + 5C + D = 0$
   c) $\overrightarrow{(A,B,C)} \cdot \overrightarrow{(3,-4,2)} = 0$
   d) $36x + 29y + 4z - 128 = 0$
7. $8x - y + 4z = 0$
8. $x - 56y - 20z + 103 = 0$
9. $2x - y + z = 3$
10. $2y - z = 0$
11. $z = 0, x = 0, y = 0$
12. $8x - 33y - 7z + 47 = 0$
13. $x + 2z = 4$
14. $2x + 8y + 13z = 5$
15. $\vec{r} = \overrightarrow{(1,0,-1)} + k\overrightarrow{(0,3,1)} + s\overrightarrow{(4,8,-1)}$
16. $25x + 3y + 6z = 15$
17. $7x - 2y - 13z + 49 = 0$
18. $\frac{4}{7}$
20. $-\frac{14}{5}$
21. $x - 4y + 3z = 16$
22. $26°$
23. $x - y - 2z - 7 = 0$
24. a) $62°$
    b) $66°$
25. $x - z = -3$
26. $14x + 16y + 3z + 47 = 0$
27. $14x - 19y - 4z + 58 = 0$
28. $k = \frac{9}{7}, m = -\frac{29}{7}$
29. $19x - 7y - 22z = 26$
30. 6
31. $5x - 2y + 11z = 0$
32. b) $6x - 7y + 2z + 11 = 0$
33. $2x - y = 0$
34. $4y + z - 7 = 0$
35. $2x - y + z = 6$
36. a) $A = B$
    b) $B = 0$
    c) $-7A + 18B + 11C + D$
    d) $A = B = 0$
    e) $4A - B + 3C = 0$
    f) $\frac{A}{4} = \frac{B}{-1} = \frac{C}{3}$
37. a) $\cos\theta =$
    $$\frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{(A_1^2 + B_1^2 + C_1^2)(A_2^2 + B_2^2 + C_2^2)}}$$

6.3 Exercises, page 262

1. a) point $(1,10,4)$
   b) see d)
   c) point $(2,2,-7.5)$
   d) infinity of solutions:
   line lies in plane
   e) point $(3,5,-1)$
   f) g) no solution: line parallel
   to, but not in, plane
3. b) $(1,3,5)$
   c) $\sqrt{14}$
4. $-2x + 3y - z = 3$
5. $-\frac{1}{6}$
6. $\vec{r} = \overrightarrow{(4,0,0)} + k\overrightarrow{(0,2,0)}$
7. a) $x = 2 + 3k$
   $y = 3 - k$
   $z = 2k$
   b) $(5,2,2)$
   c) $\sqrt{14}$
8. $(-6,4,1)$
10. $(2,-3,0)$

6.4 Exercises, page 266

1. a) intersect
   b) parallel, distinct
   c) parallel, distinct
   d) parallel, identical
   e) intersect
   f) intersect

2. a) $\begin{cases} x = \dfrac{20 - 19t}{13} \\ y = \dfrac{14 + 14t}{13} \\ z = t \end{cases}$

e) $\begin{cases} x = \dfrac{5 - t}{5} \\ y = \dfrac{1 - t}{3} \\ z = t \end{cases}$

f) $\begin{cases} x = \dfrac{2t - 1}{7} \\ y = \dfrac{-50t + 25}{14} \\ z = t \end{cases}$

3. a) $\dfrac{3x - 20}{-19} = \dfrac{13y - 14}{14} = \dfrac{z}{1}$

e) $\dfrac{5x - 5}{-1} = \dfrac{3y - 1}{-1} = \dfrac{z}{1}$

f) $\dfrac{7x + 1}{14} = \dfrac{14y - 25}{-350} = \dfrac{z}{7}$

4. $13x + 4y - 10z + 21 = 0$

5. $-6x - 5y - 4z + 13 = 0$

6. a) $\begin{cases} x = 2t + 1 \\ y = t - 3 \\ z = 5t \end{cases}$

b) $(0,-3.5,-2.5)$, $(7,0,15)$, $(1,-3,0)$

8. $\vec{r} = \overrightarrow{(-3,0,1)} + t\overrightarrow{(2,4,-3)}$

9. a) $k = 4$

b) $k = -2.5$

13. $k = 7$

14. a) $\begin{cases} x = 2t + 1 \\ y = -t - 3 \\ z = 3t + 2 \end{cases}$

c) $k = 3$, $m = 11$
$23x - 26y - 14z + 17 = 0$

6.5 Exercises, page 274

1. a) ① ③ parallel, and distinct
② not parallel

b) ① ② identical,
③ parallel and distinct

c) identical

d) parallel and distinct

2. a) point $(2,1,-1)$

b) intersect in a line

c) point $(3,-2,-1)$

d) intersect as a triangular prism

e) point $\left(-2,1,\dfrac{1}{2}\right)$

f) intersect in a line

3. a) consistent, independent

b) consistent, dependent

c) consistent, independent

d) inconsistent

e) consistent, independent

f) consistent, independent

4. $k \neq -\dfrac{69}{19}$

5. 8

6. 3

7. $Ax + By + Cz + D = 0$,
where $B = -2A$, $C = 3A$, $D \neq -5A$

8. a) line

b) triangular prism

c) point

9. a) $m \neq \pm 1$

b) $m = -1$

c) $m = 1$

10. a) linearly independent

b) coplanar

c) coplanar

6.6 Exercises, pages 278–279

1. a) $\dfrac{7}{\sqrt{35}}$

b) $\dfrac{22}{\sqrt{29}}$

c) $\dfrac{5}{\sqrt{10}}$

2. a) $\dfrac{71}{\sqrt{105}}$

b) $\dfrac{10}{\sqrt{200}}$

c) $\dfrac{88}{\sqrt{630}}$

3. a) $\dfrac{\sqrt{195}}{\sqrt{14}}$

b) $\dfrac{\sqrt{650}}{5}$

c) $\dfrac{\sqrt{932}}{\sqrt{38}}$

d) $\dfrac{\sqrt{329}}{\sqrt{13}}$

4. b) $\dfrac{7}{\sqrt{29}}$

5. a) $\dfrac{10}{\sqrt{21}}$

b) $\dfrac{7}{\sqrt{38}}$

c) $\dfrac{9}{\sqrt{44}}$

6. b) $\dfrac{\sqrt{200}}{\sqrt{18}}$

7. a) $\dfrac{\sqrt{131}}{\sqrt{70}}$

b) $\dfrac{\sqrt{4418}}{\sqrt{66}}$

8. -2 or $-\dfrac{2}{3}$

9. 3 or -15

10. $\dfrac{10}{\sqrt{2}}$

11. $\dfrac{3}{\sqrt{5}}$

12. $(3 \pm 4\sqrt{6},0,0)$

13. a) $\dfrac{13}{\sqrt{18}}$

b) $\left(\dfrac{44}{9},\dfrac{41}{18},-\dfrac{103}{18}\right)$

14. a) $\vec{r} = \overrightarrow{(-4,1,0)} + k\overrightarrow{(4,0,5)}$

b) $\sqrt{\dfrac{210}{41}}$

c) $\dfrac{1}{2}\sqrt{210}$

15. a) $5x - 13y - 4z + 33 = 0$

b) $\dfrac{33}{\sqrt{210}}$

c) $\dfrac{11}{2}$

16. 55.69 m

Inventory, page 282

1. the position vector of any point on the plane;
the position vector of a given point on the plane;
vectors parallel to the plane;
parallel to;
parameters
2. $(1,-2,6)$, $\overrightarrow{(2,5,-7)}$, $\overrightarrow{(3,-4,1)}$
3. $(3,0,-3)$, $\overrightarrow{(2,5,9)}$, $\overrightarrow{(-4,2,2)}$
4. two; line
5. $\overrightarrow{(3,4,5)}$
6. parallel
7. in a point
8. in a line
9. a triangular prism
10. $3A + 2B + 4C = 0$
11. a point
12. the line itself
13. the distance from a point P to a plane Π;
a point on Π;
a normal to Π
14. the distance from the point P to a line L;
a point on L;
a direction vector of L

Review Exercises, pages 283–287

1. a) $\vec{r} = \overrightarrow{(1,3,6)} + k\overrightarrow{(0,2,2)} + s\overrightarrow{(1,6,5)}$
$$\begin{cases} x = 1 + s \\ y = 3 + 2k + 6s \\ z = 6 + 2k + 5s \end{cases}$$
b) $\vec{r} = \overrightarrow{(5,-1,4)} + k\overrightarrow{(2,2,4)} + s\overrightarrow{(4,-1,2)}$
$$\begin{cases} x = 5 + 2k + 4s \\ y = -1 + 2k - s \\ z = 4 + 4k + 2s \end{cases}$$
c) $\vec{r} = \overrightarrow{(1,2,3)} + k\overrightarrow{(0,3,3)} + s\overrightarrow{(1,-5,-7)}$
$$\begin{cases} x = 1 + s \\ y = 2 + 3k - 5s \\ z = 3 + 3k - 7s \end{cases}$$
2. $\vec{r} = \overrightarrow{(0,1,4)} + k\overrightarrow{(-3,4,2)} + s\overrightarrow{(2,-5,1)}$
3. $\vec{r} = \overrightarrow{(-1,0,1)} + k\overrightarrow{(2,0,-4)} + s\overrightarrow{(0,2,-1)}$
4. $\vec{r} = \overrightarrow{(0,6,1)} + k\overrightarrow{(1,2,5)} + s\overrightarrow{(3,2,-1)}$
5. $\vec{r} = \overrightarrow{(0,3,-1)} + k\overrightarrow{(4,8,-3)} + s\overrightarrow{(2,-1,5)}$
6. parallel and distinct: a) f)
parallel and identical: b) d)
7. $x + 3y + 5z = 1$
8. $\left(\frac{1}{3},1,\frac{8}{3}\right)$
9. $2x - 26y - 19z + 5 = 0$
10. $x + 17y + 13z = 5$
11. $6x + y - z = 5$
12. $y - 2z = 0$
13. $x - y + 2z + 9 = 0$
14. $\vec{r} = \overrightarrow{(0,1,0)} + k\overrightarrow{(2,3,0)} + s\overrightarrow{(0,1,2)}$
15. 21°
16. 25°
17. $4x + 3y = 0$
18. $z = 3$
19. a) point $(-5,3,2)$
b) point $(2,3,0)$
c) line lies in plane
20. $-\frac{1}{2}$
22. a) Each line has the same normal $\overrightarrow{(A,B)}$. Different values of C give different lines.
b) Each plane has the same normal $\overrightarrow{(A,B,C)}$. Different values of D give different planes.
24. $2x - 2y - z = 3$
25. $x + z = 0$;
$5x - 4y - 3z = 0$
26. $\vec{r} = \overrightarrow{(4,0,0)} + k\overrightarrow{(0,2,0)}$
27. $(4,8,3)$
28. a) parallel and distinct
b) parallel and identical
c) $$\begin{cases} x = 1 - \frac{13}{11}t \\ y = \frac{14}{11}t \\ z = t \end{cases}$$
d) $$\begin{cases} x = \frac{4}{5} - \frac{3}{5}t \\ y = t \\ z = 1 - 3t \end{cases}$$
30. $2x + 3y - z - 6 = 0$
31. $x - y - z = -6$
$2x + y - z = 1$
32. a) -30
b) $\frac{15}{7}$
33. a) parallel and distinct
b) parallel and identical
34. a) point $(4,0,-2)$
b) line $$\begin{cases} x = \frac{3t + 13}{5} \\ y = \frac{6t - 9}{5} \\ z = t \end{cases}$$
c) a triangular prism
d) point $(3,0,-3)$
35. a) consistent and independent
b) consistent and dependent
c) inconsistent
d) consistent and independent
36. a) $k \neq -3, k \neq -1$
b) $k = -3$
c) $k = -1$
37. $a = 2$
$b = 8$
$c = 7.5$
$d \in \mathbb{R}$, $d \neq 4.5$ and $d \neq 2.25$
39. a) $\frac{3}{\sqrt{14}}$
b) $\frac{10}{\sqrt{83}}$
c) $\frac{44}{\sqrt{336}}$
d) 0 (point in plane)
40. a) $\frac{\sqrt{1443}}{\sqrt{38}}$
b) $\frac{\sqrt{509}}{\sqrt{37}}$
c) $\frac{\sqrt{944}}{\sqrt{38}}$
d) $\frac{\sqrt{699}}{\sqrt{66}}$
e) $\frac{\sqrt{329}}{\sqrt{13}}$
42. $27a - 7b = 0$
43. $$\begin{cases} x = \frac{-19 + 5y}{t} \\ y = t \\ z = 0 \end{cases}$$
44. $\frac{\sqrt{1350}}{\sqrt{59}}$

45. -2 or $-\frac{2}{3}$

46. **a)** $a = 2$
b) $a \neq 2$
c) no a exists

47. $\frac{7}{3}$

48. $\vec{r} = \overrightarrow{(1,0,1)} + k\overrightarrow{(7,1,-5)}$

49. $x = \frac{11p - 7q + 6w}{33}$

$y = \frac{13p - 2q - 3w}{33}$

$z = \frac{3p - 3q + w}{11}$

50. **a)** any plane that passes through $(2,1,-2)$, for example $x - y + z = -1$
b) $x + y + z + k(2x - 3y + z) = 1 + 3k$, for all k, for example $3x - 2y + 2z = 4$
c) $x + y + z + k(2x - 3y + z) = t$, where $t \neq 1 + 3k$, for example $3x - 2y + 2z = 1$

51. $(14,6,0)$

52. $a - 2b + 10 = 0$

55. **a)** $x + y + z = 1$
b) $x - y - z = -1$
c) $71°$
d) $\frac{2}{\sqrt{3}}$
e) $\frac{1}{3}$

56. **a)** $\begin{cases} x = 2 - 7t \\ y = 1 + 5t \\ z = 3 + t \end{cases}$
b) $c = 1$
c) $9x + 16y - 17z + 17 = 0$

57. **b)** $3\vec{i} + 3\vec{j} + 4\vec{k}$
c) $x = p + 1,\ y = 3,\ z = 3p - 2$ (Answers will vary.)

58. **a)** $P(-1,2,1)$
b) $\overrightarrow{(7,-3,-5)}$
$7x - 3y - 5z + 18 = 0$
c) 1.98 units

59. **a)** $42°$
b) $\frac{x}{2} = \frac{y - \frac{1}{2}}{1} \pm \frac{z - \frac{7}{2}}{-5}$
c) $\overrightarrow{OB} = \vec{i} + 4\vec{j} + 2\vec{k}$

60. **a)** $\overrightarrow{(6,-1,3)}$
b) $x = 1 + 6t,\ y = -t,\ z = 2 - 3t$
$H(-5,1,5)$
c) $q = \frac{5}{3}$

Chapter Seven Matrices and Linear Transformations

7.1 Exercises, page 294

1. $A\ 2 \times 4$
   $B\ 3 \times 2$
   $C\ 4 \times 1$

2. a) $a_{11} = 3, a_{14} = 1, a_{23} = 5, a_{33} = 8$
   b) $a_{13} = 0, a_{32} = 0$

3. a) $x = 4, y = 2, w = 0, z = 3$
   b) $x = 3, y = -2, w = 8, z = 8$

4. a) $x = 1, y = 9, w = 5, z = 1$
   b) $a = 16, b = -3, c = 0, d = -12$
   c) $a = 17, b = -\frac{2}{3}, c = \frac{8}{3}, d = -29$
   d) $x = 3, y = 2, z = 0$

5. a) $\begin{bmatrix} -2 & -3 \\ 1 & 1 \end{bmatrix}$
   b) $\begin{bmatrix} -15 & 0 \\ 9 & 0 \end{bmatrix}$
   c) $\begin{bmatrix} -17 & -3 \\ 10 & 1 \end{bmatrix}$
   d) $\begin{bmatrix} 4 & 12 \\ -2 & 5 \end{bmatrix}$
   e) $\begin{bmatrix} -\frac{33}{2} & -18 \\ \frac{19}{2} & -12 \end{bmatrix}$
   f) $\begin{bmatrix} 18 & 24 \\ -10 & 10 \end{bmatrix}$
   g) $\begin{bmatrix} 18 & 24 \\ -10 & 10 \end{bmatrix}$
   h) $\begin{bmatrix} -30 & 0 \\ 18 & 0 \end{bmatrix}$
   i) $\begin{bmatrix} -30 & 0 \\ 18 & 0 \end{bmatrix}$

6. a) $\begin{bmatrix} -\frac{5}{2} & 0 \\ \frac{3}{2} & 0 \end{bmatrix}$
   b) $\begin{bmatrix} 6 & 9 \\ -3 & -3 \end{bmatrix}$
   c) $\begin{bmatrix} 7 & 3 \\ -4 & -1 \end{bmatrix}$
   d) $\begin{bmatrix} -1 & -\frac{9}{2} \\ \frac{1}{2} & -3 \end{bmatrix}$

7. $x = 2, y = 5$
   contradicts
   $x - y = 4$

8. Answers will vary.

7.2 Exercises, page 301

1. F, G, T: yes
   H, R, S: no

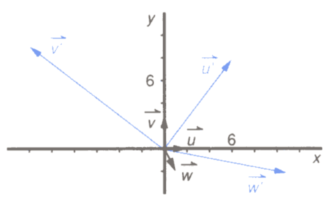
2. $F = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$
   $G = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
   $T = \begin{bmatrix} 0 & 3 \\ -1 & 0 \end{bmatrix}$

4. a) $\begin{bmatrix} 3 \\ 14 \end{bmatrix}$
   b) $\begin{bmatrix} 2 \\ -6 \end{bmatrix}$

5. a) $x = -\frac{3}{2}, y = -\frac{5}{2}$
   b) $x = 2, y = 1$
   c) $x = y = 0$
   d) $x = \pm 3, y = \pm 1$, z can have any value

6. $x^2 + y^2 = 0 \Rightarrow x = y = 0$
   contradicts
   $3x + 5y = -1$

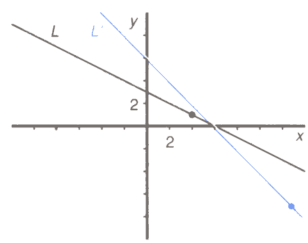
7. $M\vec{u} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$
   $M\vec{v} = \begin{bmatrix} -12 \\ 9 \end{bmatrix}$
   $M\vec{w} = \begin{bmatrix} 11 \\ -2 \end{bmatrix}$



8. a) $\begin{bmatrix} 18 \\ 24 \end{bmatrix}$
   b) $\begin{bmatrix} 17 \\ 11 \end{bmatrix}$
   c) $\begin{bmatrix} 10 \\ 66 \end{bmatrix}$
   d) $\begin{bmatrix} 174 \\ -102 \end{bmatrix}$
   so $M(N\vec{v}) \neq N(M\vec{v})$

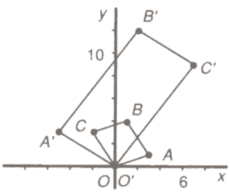
9. a) $M\vec{0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
   b) $M\vec{i} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$
   c) $M\vec{j} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$
   so $M\vec{0} = \vec{0}$
   $M\vec{i}$ = first column of M
   $M\vec{j}$ = second column of M

10. a) $A\vec{r}_0 = \begin{bmatrix} 13 \\ -7 \end{bmatrix}$
    $A\vec{m} = \begin{bmatrix} 5 \\ -5 \end{bmatrix}$
    b)



11. For example,
    a) $A(0,1)$ $B(1,3)$
    b) $A'(5,0)$ $B'(12,2)$
    c) $y = \frac{2}{7}x - \frac{10}{7}$
    d) L'

12. **a)** $C(-2,3)$
b) $A'(-5,3)$
$B'(2,12)$
$C'(7,9)$
$O'(0,0)$
c) a parallelogram

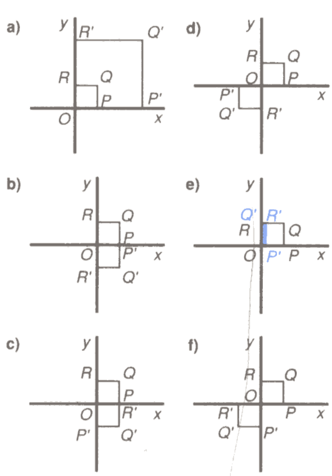


7.3 Exercises, pages 306–307

2. I leaves plane unchanged
$0_{2\times2}$ maps plane to O

3. **a)** $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$
b) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
c) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$
d) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$
e) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$
f) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

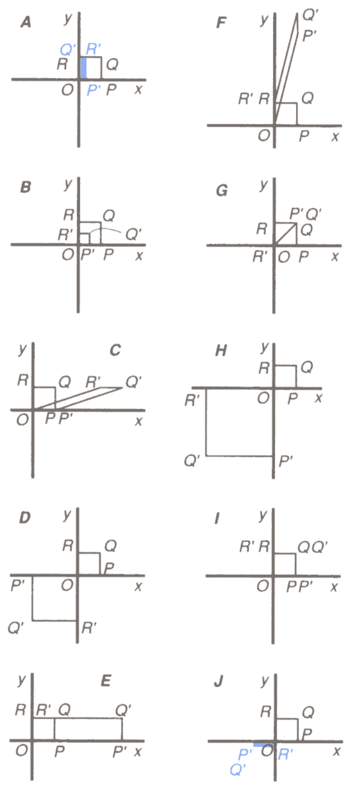
4.



5. same transformation

6. A: projection onto y-axis
B: dilation $\times \frac{1}{2}$
C: shear$\|x$-axis of factor 3
D: dilatation $\times -2$
E: stretch$\|x$-axis of factor 4
F: shear$\|y$-axis of factor 4
G: maps to line $y = x$
H: dilatation $\times -3$ and reflection in $y = x$
I: identity
J: projection onto x-axis and reflection in y-axis

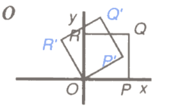
7.



8. O

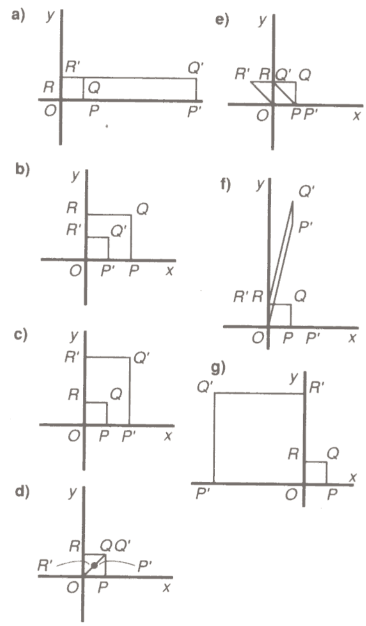
9. **a)** $(1,0) \to (1,0)$
$(a,0) \to (a,0)$
b) remains unchanged
c) $(0,1) \to (2,1)$
$(a,1) \to (a+2,1)$
d) moves 2 to the right
e) $(0,b) \to (2b,b)$
f) if $b > 0$: moves to the right, by a factor of $2b$
if $b < 0$: moves to the left, by a factor of $2b$

10. counterclockwise rotation by $30°$, about O



11. **a)** $\begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}$
b) $\begin{bmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{4} \end{bmatrix}$
c) $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$
d) $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$
e) $\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$
f) $\begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$
g) $\begin{bmatrix} -4 & 0 \\ 0 & 4 \end{bmatrix}$

12.



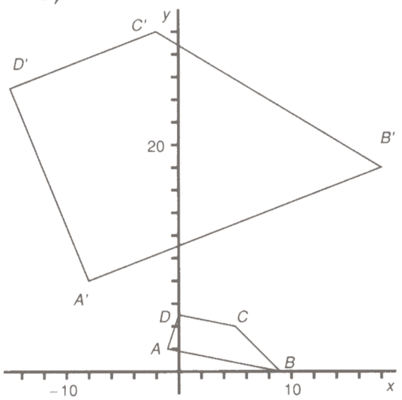
7.3 Exercises, pages 306–307, continued

13. $\begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{bmatrix}$ or $\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$

14. a) slope AB = slope $DC = -\frac{1}{5}$

$AB \| DC$

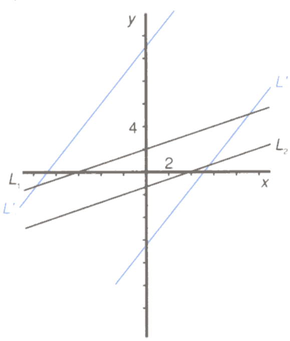
b)



c) slope $A'B'$ = slope $D'C' = \frac{5}{13}$

$A'B' \| D'C'$

15. a) direction vectors collinear



b) $\begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 8 \end{bmatrix}$

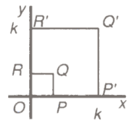
$\begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

$\begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 10 \end{bmatrix}$

c) new direction vectors

$\begin{bmatrix} 8 \\ 10 \end{bmatrix}$ and $\begin{bmatrix} -8 \\ -10 \end{bmatrix}$

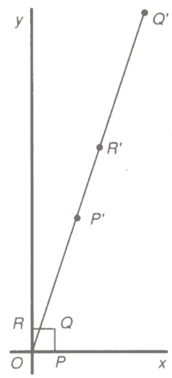
16. a) dilatated by k



b) $s\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} kx \\ ky \end{bmatrix} = k\begin{bmatrix} x \\ y \end{bmatrix}$

c) entire plane dilatated by k

17. a)



b) $S\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x+3y \\ 6x+9y \end{bmatrix} = \begin{bmatrix} a \\ 3a \end{bmatrix}$

c) entire plane mapped to line $y = 3x$

7.4 Exercises, pages 313–314

1. a) $\begin{bmatrix} 0.77 & -0.64 \\ 0.64 & 0.77 \end{bmatrix}$

b) $\begin{bmatrix} 0.17 & -0.98 \\ 0.98 & 0.17 \end{bmatrix}$

c) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

d) $\begin{bmatrix} -0.34 & -0.94 \\ 0.94 & -0.34 \end{bmatrix}$

e) $\begin{bmatrix} -0.94 & 0.34 \\ -0.34 & -0.94 \end{bmatrix}$

f) $\begin{bmatrix} 0.95 & 0.31 \\ -0.31 & 0.95 \end{bmatrix}$

2. a) $\begin{bmatrix} 0.77 & 0.64 \\ 0.64 & -0.77 \end{bmatrix}$

b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

c) $\begin{bmatrix} -0.94 & -0.34 \\ -0.34 & 0.94 \end{bmatrix}$

3. b) det(1a)) = 1
det(2a)) = −1

4. b) det(1e)) = 1
det(2c)) = −1

5. a) $\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$

b) $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

c) $\begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$

6. a) $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$

b) $\begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

c) $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$

7. $R_{300} = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} = R_{-60}$

8. $M_{30} = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix} = M_{210}$

9. $\det(I) = 1$
same area and orientation
$\det(0_{2\times 2}) = 0$
zero area, orientation not defined

10.

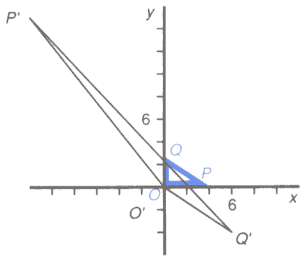
	det.	a.s.f.	orientation
A	0	0	undefined
B	0.25	0.25	same
C	1	1	same
D	4	4	same
E	4	4	same
F	1	1	same
G	0	0	undefined
H	−9	9	reversed
I	1	1	same
J	0	0	undefined

11. a) $k = 4$
b) $k = 1$
c) $k = \frac{3}{2}$

12. $\det(S) = 1$
same area and orientation

13. $\det(M) = 0$
area zero

14.



a) 3
b) $O'(0,0)$
$P'(-12,15)$
$Q'(6,-4)$
c) 21
d) OPQ and $O'P'Q'$ have reverse orientation

15. a) $\overrightarrow{OP} \cdot \overrightarrow{OQ} = 0$
b) $\overrightarrow{O'P'} \cdot \overrightarrow{O'Q'} = -132$
$\measuredangle P'O'Q' = 162°$
c) no

17. a) $\cos\theta = \frac{4}{5}$, $\sin\theta = \frac{3}{5}$

b) $\begin{bmatrix} \frac{4}{5} & -\frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{bmatrix}$

c) $\begin{bmatrix} \frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & -\frac{4}{5} \end{bmatrix}$

18. K: rotation through 37°
L: rotation through −60° (or 300°)
M: reflection in line

$$y = \left(\tan \frac{1}{2}\theta\right)x,$$

where $\theta = -37°$

that is, line $y = -\frac{1}{3}x$

N: rotation through 67° + 180°, that is, 247°

7.5 Exercises, page 318

1. A, G

2. $B^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$C^{-1} = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$

$D^{-1} = \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{bmatrix}$

$E^{-1} = \begin{bmatrix} \frac{1}{4} & 0 \\ 0 & 1 \end{bmatrix}$

$F^{-1} = \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix}$

$H^{-1} = \begin{bmatrix} 0 & -\frac{1}{3} \\ -\frac{1}{3} & 0 \end{bmatrix}$

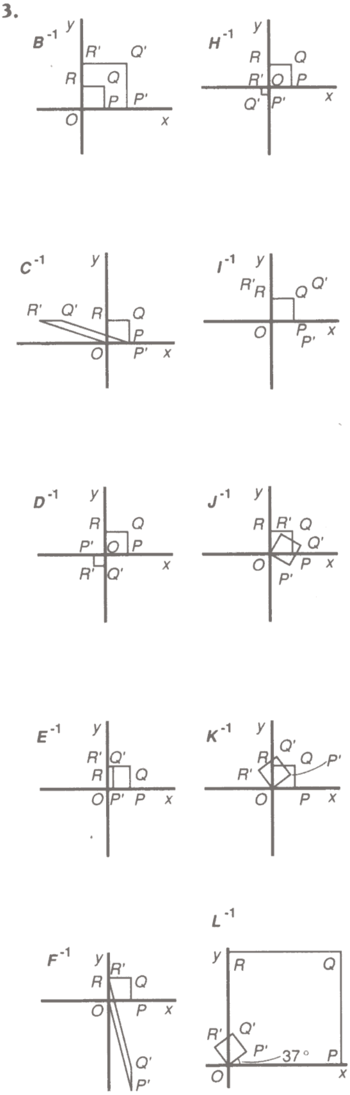
$I^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$J^{-1} = \begin{bmatrix} \cos 30° & \sin 30° \\ -\sin 30° & \cos 30° \end{bmatrix}$

$K^{-1} = \begin{bmatrix} \frac{4}{5} & -\frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{bmatrix}$

$L^{-1} = \frac{1}{25}\begin{bmatrix} 4 & -3 \\ 3 & 4 \end{bmatrix}$

3.



7.5 Exercises, page 318, continued

4.

	a.s.f	orientation
B^{-1}	4	same
C^{-1}	1	same
D^{-1}	$\frac{1}{4}$	same
E^{-1}	$\frac{1}{4}$	same
F^{-1}	1	same
H^{-1}	$\frac{1}{9}$	reversed
I^{-1}	1	same
J^{-1}	1	same
K^{-1}	1	same
L^{-1}	$\frac{1}{25}$	same

5. **a)** J and K

b) $R_\theta^{-1} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

the transpose of R

6. **a)** $R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

b) $R^{-1} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} =$

$\begin{bmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{bmatrix}$

7. $k = \frac{5}{2}$

8. $I^{-1} = I$

9. $S^{-1} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

horizontal shear of factor -2

10. **a)** $M^{-1} = \begin{bmatrix} 3 & -10 \\ -2 & 7 \end{bmatrix}$

b) $M\vec{v} = \begin{bmatrix} 13 \\ 4 \end{bmatrix}$

c) $M^{-1}\vec{v'} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

11. **a)** $M^{-1} = \frac{1}{17}\begin{bmatrix} 2 & 1 \\ -5 & 6 \end{bmatrix}$

b) $M\vec{v} = \begin{bmatrix} -8 \\ -1 \end{bmatrix}$

c) $M^{-1}\vec{v'} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

13. 1

15. **a)** $\begin{bmatrix} 3x - y \\ 12x - 4y \end{bmatrix} = \begin{bmatrix} a \\ 4a \end{bmatrix}$

b) $y = 4x$

c) no, $\det(S) = 0$

7.6 Exercises, pages 324–325

1. $LM = \begin{bmatrix} 14 & 13 \\ 32 & 25 \end{bmatrix}$

$ML = \begin{bmatrix} 1 & 4 \\ 26 & 38 \end{bmatrix}$

$LN = \begin{bmatrix} 2 & 5 \\ 10 & 15 \end{bmatrix}$

$NL = \begin{bmatrix} 21 & 32 \\ -2 & -4 \end{bmatrix}$

$MN = \begin{bmatrix} 26 & 20 \\ 16 & 25 \end{bmatrix}$

$NM = \begin{bmatrix} 49 & 29 \\ -8 & 2 \end{bmatrix}$

2. **a)** A, G

b) I

c) B

d) C

e) J, K

f) C, I, J, K

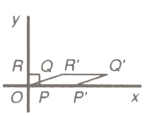
3. **a)** $IB = B, IC = C, IK = K$

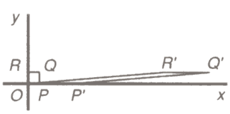
4. **a)** $CE = \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix}$

$EC = \begin{bmatrix} 4 & 12 \\ 0 & 1 \end{bmatrix}$

b) CE: stretch, then shear

EC: shear, then stretch





c) 4, for both

5. **a)** $BJ =$

$\begin{bmatrix} \frac{1}{2}\cos 30^\circ & -\frac{1}{2}\sin 30^\circ \\ \frac{1}{2}\sin 30^\circ & \frac{1}{2}\cos 30^\circ \end{bmatrix}$

$= JB$

b) rotation then dilatation = dilatation then rotation

6. **a)** $JK = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = KJ$

b) rotation 30° then rotation 60° = rotation 60° then rotation 30° = rotation 90°

7. **a)** $AK = \begin{bmatrix} 0 & 0 \\ \sin 60^\circ & \cos 60^\circ \end{bmatrix}$

$KA = \begin{bmatrix} 0 & -\sin 60^\circ \\ 0 & \cos 60^\circ \end{bmatrix}$

b) 0, for both; both singular

8. **a)** $I^2 = I$

$A^2 = A$

b) identity then identity = identity

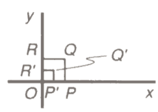
projection to y-axis then projection to y-axis = projection to y-axis

9. **a)** $B^2 = \begin{bmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{4} \end{bmatrix}$

$E^2 = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}$

b) B^2: dilatation $\times \frac{1}{4}$

E^2: stretch $\times$ 16



10. **a)** $J^2 = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

$J^3 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$J^6 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

b) rotation 30° then rotation 30° = rotation 60°

rotation 30°, then 30°, then 30° = rotation 90°

rotation 30°, six times, = rotation 180°

12. **a)** $M^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

b) $MM^{-1} = M^{-1}M =$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_{2\times 2}$

c) M and M^{-1} commute. Their product is the 2×2 identity matrix.

14. $AB = BA = I$

15. $AB = BA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_{3\times 3}$

16. $KL = \begin{bmatrix} ap + br & aq + bs \\ cp + dr & cq + ds \end{bmatrix}$

$LM = \begin{bmatrix} pw + qy & px + qz \\ rw + sy & rx + sz \end{bmatrix}$

$(KL)M =$

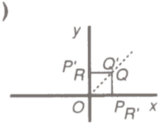
$\begin{bmatrix} apw + brw + aqy + bsy & apx + brx + aqz + bsz \\ cpw + drw + cqy + dsy & cpx + drx + cqz + dsz \end{bmatrix}$
$= K(LM)$

17. $\det(K) = ad - bc$
$\det(L) = ps - qr$
$\det(KL) = adps - adqr - bcps + bcqr$
$= (ad - bc)(ps - qr)$

18. **a)** $K^{-1} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

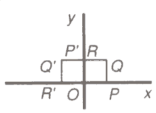
c) yes

19. **a)**



b) **c)** $M^2 = I, M^3 = M, M^4 = I$

20. **a)**



b) $M^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$
(rotation 180°)
$M^3 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$
(rotation 270°)
$M^4 = I$
(rotation 360°)

21. **a)** back to original
b) $T^2 = I$

22. **a)** rotation through 2θ
b) $R^2 = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix} =$
$\begin{bmatrix} \cos^2\theta - \sin^2\theta & -2\sin\theta\cos\theta \\ 2\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$

23. **a)** P: rotation through $\alpha = -53°$
$\left(\tan\alpha = -\frac{4}{3}\right)$
Q: rotation through $\alpha = 53°$
$\left(\tan\alpha = \frac{4}{3}\right)$
b) $PQ = I$
c) $P^{-1} = Q, Q^{-1} = P$

24. **a)** $AB = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$ $BA = \begin{bmatrix} 1 & 2 \\ 3 & 8 \end{bmatrix}$
b) $A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$
$B^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$
c) $A^{-1}B^{-1} = \begin{bmatrix} 4 & -1 \\ -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$
d) $(A^{-1}B^{-1})(AB) = \begin{bmatrix} 24 & 10 \\ -\frac{17}{2} & -\frac{7}{2} \end{bmatrix}$
$(A^{-1}B^{-1})(BA) = I$
e) $(A^{-1}B^{-1})^{-1} = BA$

7.7 Exercises, pages 331–332

1. **a)** $L^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$
$M^{-1} = \begin{bmatrix} 1 & 4 \\ 1 & 5 \end{bmatrix}$
b) $LM = \begin{bmatrix} 7 & -5 \\ 3 & -2 \end{bmatrix}$
$L^{-1}M^{-1} = \begin{bmatrix} -1 & -7 \\ 1 & 6 \end{bmatrix}$
$M^{-1}L^{-1} = \begin{bmatrix} -2 & 5 \\ -3 & 7 \end{bmatrix}$
c) $(LM)^{-1}\begin{bmatrix} -2 & 5 \\ -3 & 7 \end{bmatrix}$
d) $(LM)(M^{-1}L^{-1}) = I$
$(LM)(L^{-1}M^{-1}) \neq I$

2. $B^{-1}A^{-1} = A^{-1}B^{-1} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$
reflection in x-axis then dilatation $\times$ 2 = dilatation $\times$ 2 then reflection in x-axis

7.7 Exercises, pages 331–332, continued

3. a) $x = 4, y = \frac{7}{2}$
   b) no solution
   c) $x = k, y = \frac{2k-4}{7}$
   d) none
   e) $x = k, y = \frac{7+k}{4}$
   f) $x = \frac{3}{17}, y = \frac{32}{17}$
4. a) (only possible) $x = 4, y = \frac{7}{2}$
5. a) none
   b) $\begin{bmatrix} 2k-4 \\ k \end{bmatrix}$
   c) $\begin{bmatrix} 3+2k \\ k \end{bmatrix}$
   d) none
6. $\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} x \\ -2x+2 \end{bmatrix} =$
   $\begin{bmatrix} 2x - 2x + 2 \\ 4x - 4x + 4 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$
7. a) $L + M = \begin{bmatrix} p+w & q+x \\ r+y & s+z \end{bmatrix}$
   $KL = \begin{bmatrix} ap+br & aq+bs \\ cp+dr & cq+ds \end{bmatrix}$
   $KM = \begin{bmatrix} aw+by & ax+bz \\ cw+dy & cx+dz \end{bmatrix}$
   $K(L+M) =$
   $\begin{bmatrix} ap+br+aw+by & aq+bs+ax+bz \\ cp+dr+cw+dy & cq+ds+cx+dz \end{bmatrix}$
   $LK = \begin{bmatrix} pa+qc & pb+qd \\ ra+sc & rb+sd \end{bmatrix}$
   $MK = \begin{bmatrix} wa+xc & wb+xd \\ ya+zc & yb+zd \end{bmatrix}$
   $(L+M)K =$
   $\begin{bmatrix} pa+qc+wa+xc & pb+qd+wb+xd \\ ra+sc+ya+zc & rb+sd+yb+zd \end{bmatrix}$
8. a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
   b) no; a) is a counterexample
9. a) $PQ = RQ \Rightarrow (PQ)Q^{-1} = (RQ)Q^{-1}$
   $\Rightarrow P(QQ^{-1}) = R(QQ^{-1})$
   $\Rightarrow P = R$
   b) If $\det(Q) = 0$, P does not have to equal R.
10. no, in general
11. $\begin{bmatrix} \pm 2 & 0 \\ 0 & \pm 2 \end{bmatrix}$ or
   $\begin{bmatrix} x & y \\ \frac{4-x^2}{y} & -x \end{bmatrix}, x, y \in \mathbb{R}, y \neq 0$
12. a) $A(I - A) = I \Rightarrow I - A = A^{-1}$
   b) $A^3 = A^2A = (A - I)(A)$
   $= A^2 - A$
   $= -I$
   c) $p = 2, q = -1$
13. a) $PQ = \begin{bmatrix} 2a+4b & 5b+a \\ 4a+20b & 19b+5a \end{bmatrix}$
   $= QP$
   b) $Q = \begin{bmatrix} 0 & 1 \\ 4 & 3 \end{bmatrix}$
14. a) $R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$
   $M = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
   b) $RM = MR \Rightarrow \sin\theta = -\sin\theta$
   $\Rightarrow 2\sin\theta = 0$
   $\Rightarrow \theta \in \{0°, 180°\}$
18. b) $x = 99, y = -52, z = -36$
19. c) $y = \frac{3}{2}x$, and $y = -x$

In Search of, page 335

1. a) 5, −1
   b) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$
   c) $y = x$ and $y = -\frac{1}{2}x$
2. a) 3
   b) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$
   c) $y = -x$
   a) $\pm i$
   b) none
   c) none
4. b) $a = b$ and $h = 0$
5. a) $s\vec{v}\cdot\vec{u} = s\vec{u}\cdot\vec{v}$
   b) $p\vec{u}\cdot\vec{v} = q\vec{v}\cdot\vec{u}$
   c) Since $p \neq q$, $\vec{u}\cdot\vec{v} = 0$

Inventory, page 338

1. 4
2. $p \times q$
3. n; n
4. 6; 3; 2
5. $T\vec{u} + T\vec{v}$; $k(T\vec{v})$
6. the origin

7. parallelism

8. $\vec{i}$ by M;
image of $\vec{j}$ by M

9. $\begin{bmatrix} 0.6 & -0.8 \\ 0.8 & 0.6 \end{bmatrix}$

10. B; A

11. commutative

12. $q = r$; $p \times s$

13. $\det(M) \neq 0$;
$\dfrac{1}{\det(M)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

14. $\det(M) = 0$

15. reversed

16. singular transformation

17. $\det(M)$

18. $\det(M) > 0$; $\det(M) < 0$

Review Exercises, pages 339–341

1. a) $\begin{bmatrix} -1 & -3 \\ -2 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 18 & 0 \\ -6 & -9 \end{bmatrix}$

c) $\begin{bmatrix} -1 & 15 \\ 12 & 3 \end{bmatrix}$

d) $\begin{bmatrix} 12 & 2 \\ -4 & 4 \end{bmatrix}$

e) $\begin{bmatrix} -19 & 7 \\ 18 & -28 \end{bmatrix}$

f) $\begin{bmatrix} \frac{11}{2} & -\frac{1}{2} \\ -3 & 2 \end{bmatrix}$

g) $\begin{bmatrix} 24 & -6 \\ -22 & 39 \end{bmatrix}$

h) $\begin{bmatrix} -8 & -24 \\ -16 & 0 \end{bmatrix}$

i) $\begin{bmatrix} -8 & -24 \\ -16 & 0 \end{bmatrix}$

2. a) $\begin{bmatrix} 2 & 0 \\ -\frac{2}{3} & -1 \end{bmatrix}$

b) $\begin{bmatrix} 2 & 6 \\ 4 & 0 \end{bmatrix}$

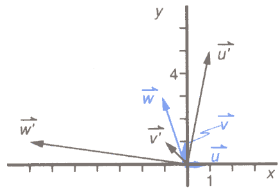
c) $\begin{bmatrix} -4 & 4 \\ 6 & -7 \end{bmatrix}$

d) $\begin{bmatrix} \frac{13}{4} & \frac{3}{4} \\ -\frac{1}{2} & -\frac{3}{2} \end{bmatrix}$

3. a) $x = 1, y = -\dfrac{11}{4}$

b) $x = y = 0$

4. $M\vec{u} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$, $M\vec{v} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$,
$M\vec{w} = \begin{bmatrix} -7 \\ 1 \end{bmatrix}$



5. a) $\begin{bmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{4} \end{bmatrix}$

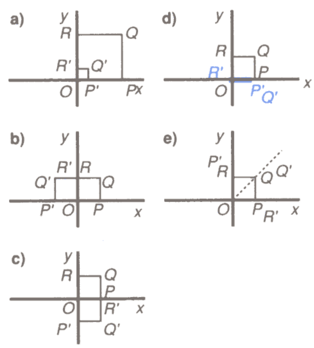
b) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

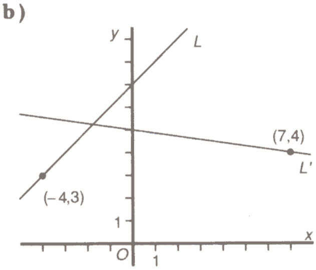
e) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

6.



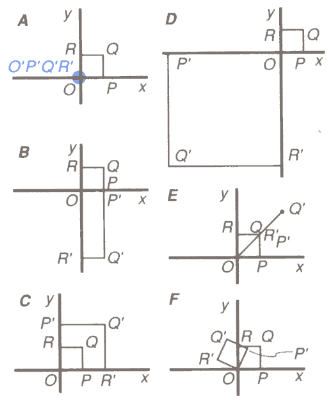
7. a) $\vec{r} = \begin{bmatrix} 7 \\ 4 \end{bmatrix} + k\begin{bmatrix} 7 \\ -1 \end{bmatrix}$

b)



8. A: null transformation
B: stretch $\parallel$ y-axis of factor 3, and reflection in x-axis
C: reflection in $y = x$ and dilatation $\times$ 2
D: reflection in origin and dilatation $\times$ 5
E: mapping to $y = x$
F: counterclockwise rotation of 65° about origin

9.



10. a)

	a.s.f.	orientation
A	0	undefined
B	3	reversed
C	4	reversed
D	25	same
E	0	undefined
F	1	same

b) A and E

c) $B^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & -\frac{1}{3} \end{bmatrix}$

$C^{-1} = \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$

$D^{-1} = \begin{bmatrix} -\frac{1}{5} & 0 \\ 0 & -\frac{1}{5} \end{bmatrix}$

$F^{-1} = \begin{bmatrix} \cos 65^\circ & \sin 65^\circ \\ -\sin 65^\circ & \cos 65^\circ \end{bmatrix}$

Review Exercises, pages 339–341, continued

11. a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 0 & -10 \\ -10 & 0 \end{bmatrix}$

c) $\begin{bmatrix} -5 & -5 \\ -5 & -5 \end{bmatrix}$

d) $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

12. a) $\begin{bmatrix} 0.64 & -0.77 \\ 0.77 & 0.64 \end{bmatrix}$

b) $\begin{bmatrix} -0.57 & -0.82 \\ 0.82 & -0.57 \end{bmatrix}$

c) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

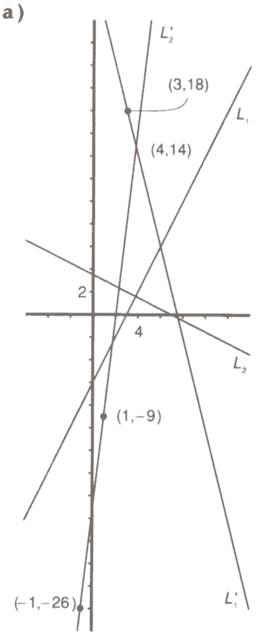
13. a) $\begin{bmatrix} -0.77 & 0.64 \\ 0.64 & 0.77 \end{bmatrix}$

b) $\begin{bmatrix} -0.10 & -0.99 \\ -0.99 & 0.10 \end{bmatrix}$

c) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

14. $R_{225} = R_{-135}$, same rotation

16. a)



b) L_1': $\vec{r} = \begin{bmatrix} 3 \\ 18 \end{bmatrix} + k\begin{bmatrix} 1 \\ -4 \end{bmatrix}$

L_2': $\vec{s} = \begin{bmatrix} -1 \\ -26 \end{bmatrix} + p\begin{bmatrix} 2 \\ 17 \end{bmatrix}$

c) no

17. M: rotation through α,

$\tan \alpha = \frac{y}{x}$

N: reflection in $y = \left(\tan\frac{1}{2}\alpha\right)x$,

$\tan \alpha = \frac{y}{x}$

18. a) $M_\theta = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$

b) $M_\theta^{-1} = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$

c) $M_\theta^{-1} = M_\theta$

19. $\begin{bmatrix} 2y \\ y \end{bmatrix}$ (or $\begin{bmatrix} 2k \\ k \end{bmatrix}$)

20. a) $A^{-1} = A$, $B^{-1} = B$, $C^{-1} = C$

b) Reflections are self-inverses.

21. a) Plane is brought back to original status.

b) I

22. a) $AB = \begin{bmatrix} 1 & -3 \\ -10 & -6 \end{bmatrix}$

$BA = \begin{bmatrix} 8 & -4 \\ 17 & -13 \end{bmatrix}$

b) $\det(A) = -3$

$\det(B) = 12$

$\det(AB) = -36 = \det(BA)$

23. no, for example

$\begin{bmatrix} 2 & 0 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, but

$\begin{bmatrix} 0 & 0 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 14 & 0 \end{bmatrix}$

24. a) yes

b) no (sometimes many, sometimes none)

25. $\begin{bmatrix} 0 & bt \\ -ct & 0 \end{bmatrix}$

27. a) $N = \dfrac{1}{4-4p}\begin{bmatrix} 2 & -p \\ -4 & 2 \end{bmatrix}$

$p = 1$

c) $p = -2$, $q = -1$

d) $p = -1$ or $p = 3$

28. a) sketch not provided

b) $\lambda = \dfrac{33}{14}$ $\mu = \dfrac{16}{7}$

d) $D\left(8, \frac{1}{2}\right)$

$|\overrightarrow{BA}| = \sqrt{17}$, $|\overrightarrow{BC}| = \dfrac{3\sqrt{17}}{2}$

e) $A'(3,-4)$ $B'(-2,5)$

$C'\left(\frac{5}{2}, 2\right)$ $D'\left(\frac{15}{2}, -7\right)$

$\alpha = -2$

29. C

30. $\begin{bmatrix} 7 \\ 0 \end{bmatrix}$

31. a) $O'(0,0)$ $A'(1,2)$ $B'(4,3)$ $C'(3,1)$

d) shear with invariant line $y = 2x$; of factor 1

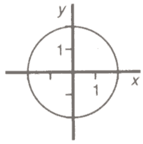
e) image of A under T^{-1}: $(1,2)$

image of C under T^{-1}: $(1,-3)$

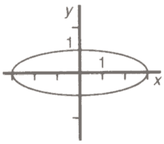
Chapter Eight
Transformations of Conics

8.1 Exercises, page 348

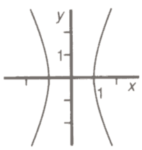
1. a)



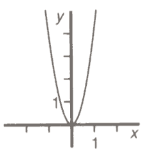
b)



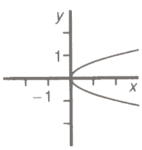
c)



d)



e)



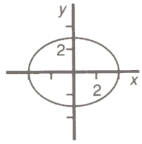
2. Circles as follows

	centre	radius
a)	(0,0)	4
b)	(0,0)	3
c)	(0,0)	2
d)	(0,0)	$\sqrt{5}$
e)	(0,0)	2
f)	(0,0)	$\sqrt{2}$

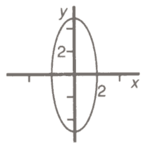
$g = f = 0$ for each

	a	b	c
a)	1	1	−16
b)	1	1	−9
c)	4	4	−16
d)	3	3	−15
e)	−2	−2	8
f)	−5	−5	10

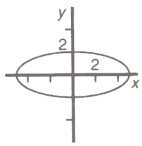
3. a)



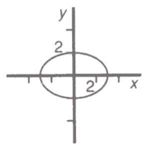
b)



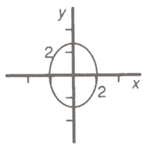
c)



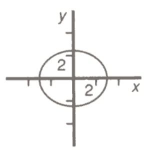
d)



e)



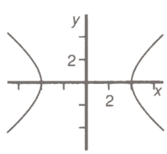
f)



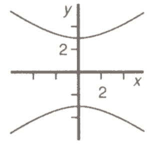
$g = f = 0$ for each

	a	b	c
a)	9	16	−144
b)	25	4	−100
c)	4	25	−100
d)	−4	−9	36
e)	−32	−18	144
f)	−2	−3	18

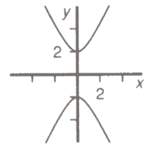
4. a)



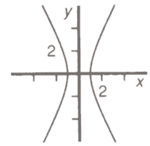
b)



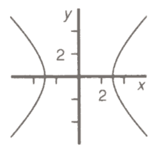
c)



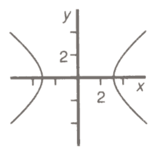
d)



e)



f)

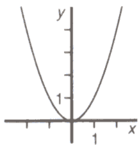


$g = f = 0$ for each

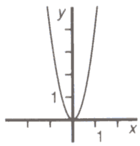
	a	b	c
a)	9	−16	−144
b)	9	−16	144
c)	4	1	4
d)	−8	2	8
e)	−1	1	9
f)	3	−5	−30

8.1 Exercises, page 348, continued

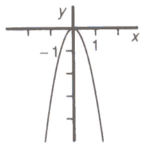
5. a)



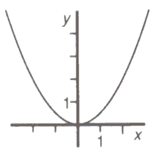
b)



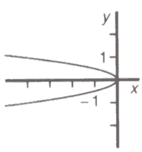
c)



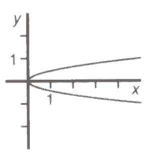
d)



e)



f)



	a	b	c	g	f
a)	1	0	0	0	$-\frac{1}{2}$
b)	4	0	0	0	$-\frac{1}{2}$
c)	3	0	0	0	$\frac{1}{2}$
d)	−1	0	0	0	1
e)	0	4	0	$\frac{1}{2}$	0
f)	0	5	0	0	$-\frac{1}{2}$

6. **a)** circle
b) hyperbola
c) parabola
d) hyperbola
e) circle
f) ellipse
g) hyperbola
h) parabola

circles
a) (0,0), 5
e) (0,0), 4

ellipses
f) (0,0), (±3,0), x-axis
g) (0,0), (0,±4), y-axis

hyperbolas
b) (0,0), (±3,0), x-axis
d) (0,0), (0,±2), y-axis

parabolas
c) (0,0), (positive) y-axis
h) (0,0), (negative) x-axis

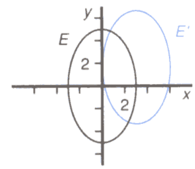
7. **a)** ellipse
b) hyperbola
c) parabola
d) circle
e) parabola
f) hyperbola
g) ellipse
h) circle
i) hyperbola
j) ellipse

8.2 Exercises, pages 354–355

1. **a)** (6,11)
b) (−3,11)
c) (5,−2)
d) (0,0)

2. **a)** $25x^2 + 9y^2 - 150x - 18y + 9 = 0$

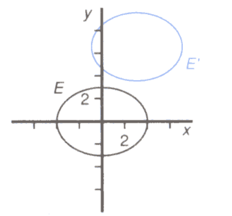
b)



c)

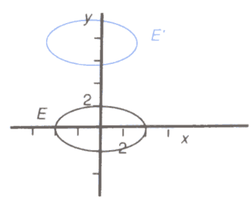
	a	b	c	g	f
E	25	9	−225	0	0, $ab > 0$
E'	25	9	9	−75	−9, $ab > 0$

3. **a)** $9x^2 + 16y^2 - 54x - 192y + 513 = 0$



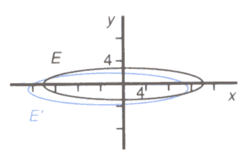
	a	b	c	g	f
E	9	16	−144	0	0, $ab > 0$
E'	9	16	513	−27	−96, $ab > 0$

b) $x^2 + 4y^2 + 2x - 56y + 181 = 0$



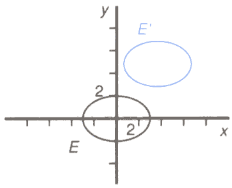
	a	b	c	g	f
E	1	4	−16	0	0, $ab > 0$
E'	1	4	181	1	−28, $ab > 0$

c) $8x^2 + 200y^2 + 48x + 800y - 728 = 0$



	a	b	c	g	f
E	8	200	−1600	0	0, $ab > 0$
E'	8	200	−728	24	400, $ab > 0$

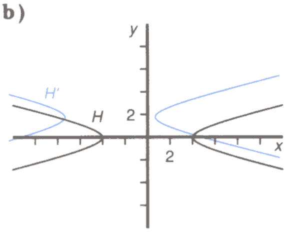
d) $4x^2 + 9y^2 - 32x - 90y + 253 = 0$



	a	b	c	g	f
E	4	9	−36	0	0, $ab > 0$
E'	4	9	253	−16	−45, $ab > 0$

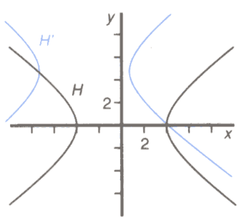
4. a) $x^2 - 16y^2 + 6x + 64y - 71 = 0$

b)



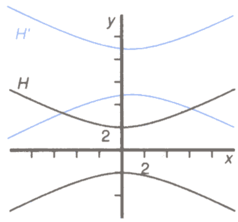
	a	b	c	g	f
H	1	−16	−16	0	0, $ab < 0$
H'	1	−16	−71	3	32, $ab < 0$

5. a) $9x^2 - 16y^2 + 54x + 160y + 463 = 0$



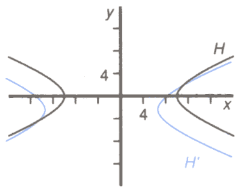
	a	b	c	g	f
H	9	−16	−144	0	0, $ab < 0$
H'	9	−16	463	27	80, $ab < 0$

b) $x^2 - 4y^2 - 2x + 56y - 179 = 0$



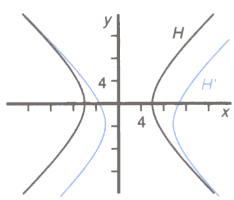
	a	b	c	g	f
H	1	−4	−16	0	0, $ab < 0$
H'	1	−4	−179	−1	28, $ab < 0$

c) $8x^2 - 50y^2 + 48x - 200y - 928 = 0$



	a	b	c	g	f
H	8	−50	−800	0	0, $ab < 0$
H'	8	−50	−928	24	−100, $ab < 0$

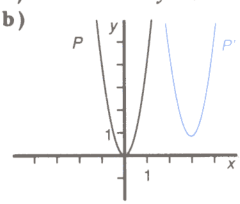
d) $x^2 - y^2 - 8x - 6y - 29 = 0$



	a	b	c	g	f
H	1	−1	−36	0	0, $ab < 0$
H'	1	−1	−29	−4	−3, $ab < 0$

6. a) $4x^2 - 24x - y + 37 = 0$

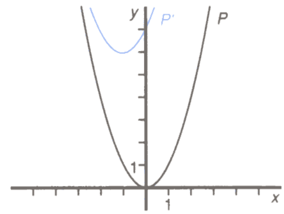
b)



c)

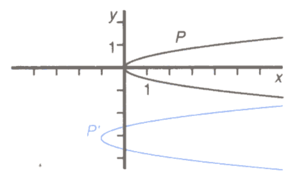
	a	b	c	g	f
P	4	0	0	0	$-\frac{1}{2}$, $ab = 0$
P'	4	0	37	−12	$-\frac{1}{2}$, $ab = 0$

7. a) $x^2 + 2x - y + 7 = 0$



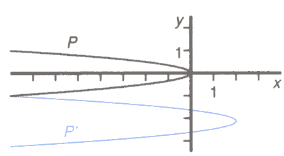
	a	b	c	g	f
P	1	0	0	0	$-\frac{1}{2}$, $ab = 0$
P'	1	0	7	1	$-\frac{1}{2}$, $ab = 0$

b) $4y^2 - x + 24y + 35 = 0$



	a	b	c	g	f
P	0	4	0	$-\frac{1}{2}$	0, $ab = 0$
P'	0	4	35	$-\frac{1}{2}$	12, $ab = 0$

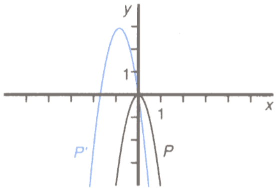
c) $8y^2 + x + 32y + 30 = 0$



	a	b	c	g	f
P	0	8	0	$\frac{1}{2}$	0,$ab = 0$
P'	0	8	30	$\frac{1}{2}$	16,$ab = 0$

8.2 Exercises, pages 354–355, continued

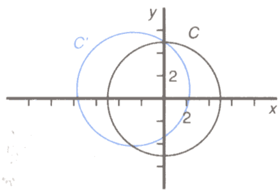
d) $4x^2 + 8x + y + 1 = 0$



	a	b	c	g	f
P	4	0	0	0	$\frac{1}{2}$, $ab = 0$
P'	4	0	1	4	$\frac{1}{2}$, $ab = 0$

8. a) $x^2 + y^2 + 6x - 2y - 15 = 0$

b)



c)

	a	b	c	g	f
C	1	1	−25	0	0, $ab > 0$
C'	1	1	−15	3	−1, $ab > 0$

9. a) $x^2 + y^2 - 6x + 2y - 134 = 0$
C: centre (0,0) radius 12
C': centre (3,−1) radius 12

	a	b	c	g	f
C	1	1	−144	0	0, $ab > 0$
C'	1	1	−134	−3	1, $ab > 0$

b) $x^2 + y^2 + 10x + 4y + 4 = 0$
C: centre (0,0) radius 5
C': centre (−5,−2) radius 5

	a	b	c	g	f
C	1	1	−25	0	0, $ab > 0$
C'	1	1	4	5	2, $ab > 0$

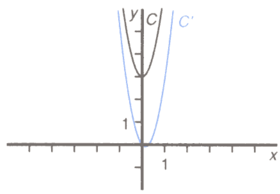
c) $8x^2 + 8y^2 - 48x + 32y + 88 = 0$
C: centre (0,0) radius $\sqrt{2}$
C': centre (3,−2) radius $\sqrt{2}$

	a	b	c	g	f
C	8	8	−16	0	0, $ab > 0$
C'	8	8	88	−24	16, $ab > 0$

d) $-4x^2 - 4y^2 + 32x + 40y - 148 = 0$
C: centre (0,0) radius 2
C': centre (4,5) radius 2

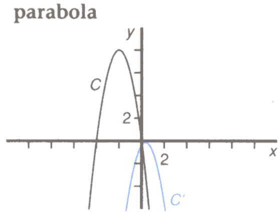
	a	b	c	g	f
C	−4	−4	16	0	0, $ab > 0$
C'	−4	−4	−148	16	20, $ab > 0$

11. a) $4x^2 - y = 0$
b) parabola
c)

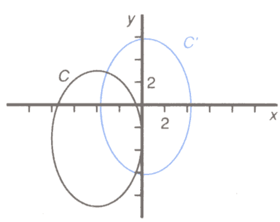


12. a) $x^2 + y^2 - 25 = 0$
C': circle centre (0,0) radius 5
C: circle centre (−2,−3) radius 5

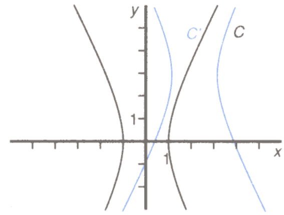
b) $2x^2 + y = 0$
parabola



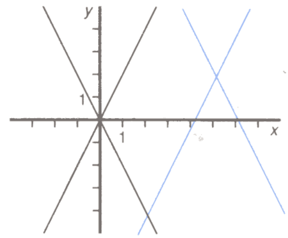
c) $9x^2 + 4y^2 - 144 = 0$
ellipse



d) $4x^2 - y^2 - 4 = 0$
hyperbola



13. a) $2x - y = 0$, $2x + y = 0$
b) $4x^2 - y^2 - 40x + 4y + 96 = 0$
c)



d) two lines, rather than a circle, ellipse, parabola, or hyperbola

14. a) $ax^2 + by^2 - 2ahx + 2bky + ah^2 + bk^2 + t = 0$

15. $(x,y) \rightarrow (x + 2, y + 3)$

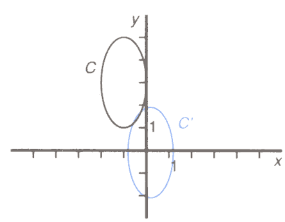
17. b) (h,k)
c) $2a$; $2b$ (if $a > b$)
$2b$; $2a$ (if $b > a$)

18. a) $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$
b) $y - k = a(x - h)^2$

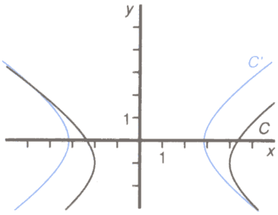
8.3 Exercises, page 360

1. ellipse: a) e)
circle: d)
hyperbola: b) f)
parabola: c)

2. a) ellipse
b) $(x,y) \rightarrow (x + 1, y - 3)$
c) $4x^2 + y^2 = 4$
d)

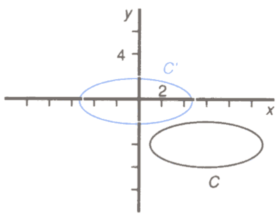


3. a) hyperbola
$(x,y) \to (x-1, y+1)$
$4x^2 - 9y^2 = 36$

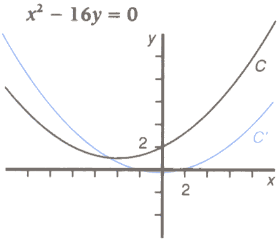


b) circle
$(x,y) \to (x-3, y+5)$
$x^2 + y^2 = 36$
C: circle centre $(3,-5)$
radius 6
C': circle centre $(0,0)$
radius 6

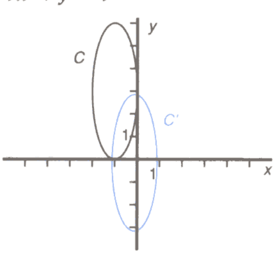
c) ellipse
$(x,y) \to (x-6, y+4)$
$4x^2 + 25y^2 = 100$



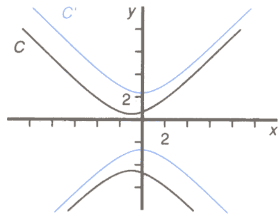
d) parabola
$(x,y) \to (x+4, y-1)$
$x^2 - 16y = 0$



e) ellipse
$(x,y) \to (x+1, y-3)$
$9x^2 + y^2 = 9$

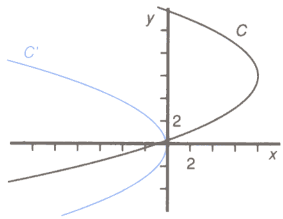


f) hyperbola
$(x,y) \to (x+1, y+2)$
$4x^2 - 4y^2 = -25$



g) circle
$(x,y) \to (x+2, y+3)$
$2x^2 + 2y^2 = 25$
C: circle centre $(-2,-3)$
radius $\sqrt{\frac{25}{2}}$
C': circle centre $(0,0)$
radius $\sqrt{\frac{25}{2}}$

h) parabola
$(x,y) \to (x-8, y-6)$
$y^2 + 4x = 0$



4. a) $(x,y) \to (x-1, y+2)$
b) $4x^2 - y^2 = 0$;
$2x - y = 0,\ 2x + y = 0$
c) $2x + y = 0,\ 2x - y - 4 = 0$
d) $ab < 0$, but graph is two lines

6. b) $g^2 + f^2 < c$
c) $g^2 + f^2 = c$

8. a) parabola

8.4 Exercises, page 364

1. a) $[46 \quad 52]$
b) $\begin{bmatrix} 14 \\ 23 \end{bmatrix}$

2. a) $7x^2 + 16xy + 9y^2 = k$
b) $6x^2 - y^2 = k$
c) $4x^2 - 10xy + 2y^2 = k$
d) $-2x^2 - 8xy + 3y^2 = k$
e) $12x^2 + 20xy + 2y^2 = k$
f) $-2x^2 + 18xy + y^2 = k$

3. a) $\begin{bmatrix} 4 & 3 \\ 3 & 5 \end{bmatrix}$, $[3]$
b) $\begin{bmatrix} 7 & -4 \\ -4 & 3 \end{bmatrix}$, $[1]$
c) $\begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}$, $[9]$
d) $\begin{bmatrix} 5 & -1.5 \\ -1.5 & -7 \end{bmatrix}$, $[8]$
e) $\begin{bmatrix} 9 & 0 \\ 0 & -11 \end{bmatrix}$, $[-5]$
f) $\begin{bmatrix} 1 & -0.5 \\ -0.5 & 5 \end{bmatrix}$, $[3]$

4. a) $[x \quad y]\begin{bmatrix} 4 & 3 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = [3]$
b) $[x \quad y]\begin{bmatrix} 7 & -4 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = [1]$
c) $[x \quad y]\begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = [9]$
d) $[x \quad y]\begin{bmatrix} 5 & -1.5 \\ -1.5 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = [8]$
e) $[x \quad y]\begin{bmatrix} 9 & 0 \\ 0 & -11 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = [-5]$
f) $[x \quad y]\begin{bmatrix} 1 & -0.5 \\ -0.5 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = [3]$

5. a) $\begin{bmatrix} 7 & 8 \\ 8 & 9 \end{bmatrix}$
b) $\begin{bmatrix} 6 & 0 \\ 7 & -1 \end{bmatrix}$
c) $\begin{bmatrix} 3 & 5 \\ 1 & 4 \end{bmatrix}$
d) $\begin{bmatrix} -2 & -4 \\ -4 & 3 \end{bmatrix}$
e) $\begin{bmatrix} 12 & 10 \\ 0 & 2 \end{bmatrix}$
f) $\begin{bmatrix} 1 & 7 \\ 4 & 1 \\ 3 & 2 \end{bmatrix}$

6. a) $\begin{bmatrix} 11 & 4 \\ 19 & 8 \end{bmatrix}$
b) $\begin{bmatrix} 11 & 19 \\ 4 & 8 \end{bmatrix}$
c) $\begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}, \begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} 11 & 19 \\ 4 & 8 \end{bmatrix}$

7. a) $(AB)^t = B^tA^t = \begin{bmatrix} 3 & 4 \\ 10 & 17 \end{bmatrix}$
b) $(AB)^t = B^tA^t = \begin{bmatrix} -15 & -2 \\ -8 & 8 \end{bmatrix}$
c) $(AB)^t = B^tA^t = \begin{bmatrix} 14 & 8 \\ 21 & 8 \end{bmatrix}$

8.4 Exercises, page 364, continued

8. **a)** $\begin{bmatrix} 0.87 & -0.50 \\ 0.50 & 0.87 \end{bmatrix}$

b) $\begin{bmatrix} 0.94 & -0.34 \\ 0.34 & 0.94 \end{bmatrix}$

c) $\begin{bmatrix} -0.77 & -0.64 \\ 0.64 & -0.77 \end{bmatrix}$

d) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

9. **a)** $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

b) $\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$

c) $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

10. $\begin{bmatrix} 3.06 & -5.79 \\ 2.57 & 6.89 \end{bmatrix}$

11. **a)** $\begin{bmatrix} 2.10 & -1.63 \\ 2.37 & 4.83 \end{bmatrix}$

b) $\begin{bmatrix} -0.10 & -0.23 \\ 5.83 & 3.60 \end{bmatrix}$

12. **a)** $\begin{bmatrix} \frac{3\sqrt{3}}{2} - \frac{1}{2} & \frac{\sqrt{3}}{2} - \frac{5}{2} \\ \frac{3}{2} + \frac{\sqrt{3}}{2} & \frac{1}{2} + \frac{5\sqrt{3}}{2} \end{bmatrix}$

b) $\begin{bmatrix} \frac{5}{2} - \frac{3\sqrt{3}}{2} & \frac{3}{2} - \sqrt{3} \\ \frac{5\sqrt{3}}{2} + \frac{3}{2} & \frac{3\sqrt{3}}{2} + 1 \end{bmatrix}$

13. **b)** $R^tR = R^{-1}R = I$

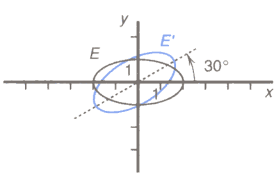
8.5 Exercises, page 369

1. **a)** $[x \;\; y]\begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = [4]$

b) $[x \;\; y]\begin{bmatrix} 1.8 & -1.3 \\ -1.3 & 3.3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = [4]$

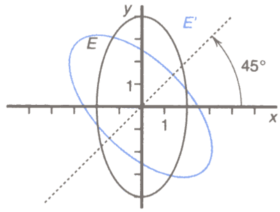
c) $1.8x^2 - 2.6xy + 3.3y^2 = 4$

d)

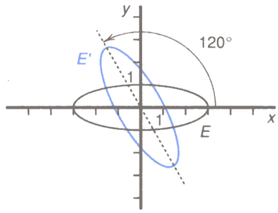


2. **a)** $2.5x^2 + 3xy + 2.5y^2 = 16$

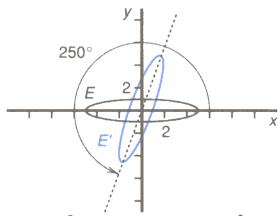
b)



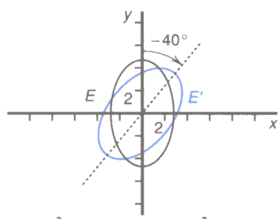
3. **a)** $2x^2 + 6.9xy + 3y^2 = 9$



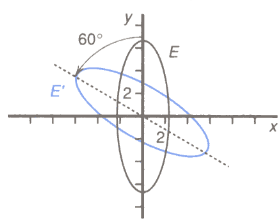
b) $22.5x^2 - 13.5xy + 6.5y^2 = 100$



c) $18.4x^2 - 15.8xy + 15.6y^2 = 200$



d) $1.5x^2 + 1.7xy + 2.5y^2 = 4$



5. **a)** $\frac{5}{2}x^2 + 3xy + \frac{5}{2}y^2 = 16$

See 2b) for sketch.

b) $3x^2 - 4\sqrt{3}xy + 7y^2 = 9$

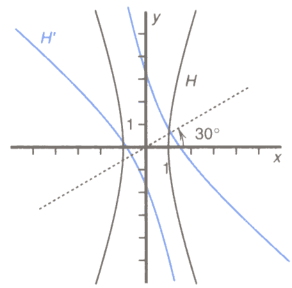
See 3a) for sketch.

c) $\frac{3}{2}x^2 + \sqrt{3}xy + \frac{5}{2}y^2 = 4$

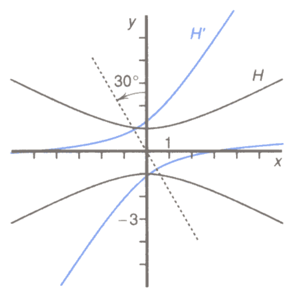
See 3d) for sketch.

6. **a)** $6.5x^2 + 8.6xy - 1.5y^2 = 9$

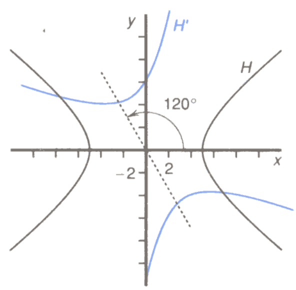
b)



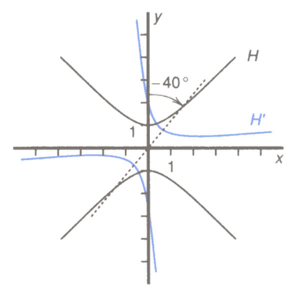
7. **a)** $-0.3x^2 + 4.3xy - 2.8y^2 = -4$



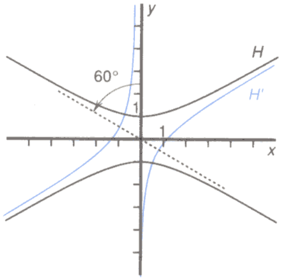
b) $-14.8x^2 - 35.5xy + 5.8y^2 = 400$



c) $-0.2x^2 - 2.0xy + 0.2y^2 = -1$



d) $-2x^2 + 3.5xy = -3$



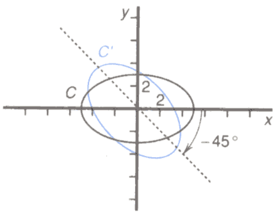
9. a) $\frac{13}{2}x^2 + 5\sqrt{3}xy - \frac{3}{2}y^2 = 9$
See 6b) for sketch.

b) $-\frac{59}{4}x^2 - \frac{41\sqrt{3}}{2}xy + \frac{23}{4}y^2 = 400$
See 7b) for sketch.

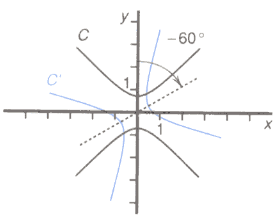
c) $-2x^2 + 2\sqrt{3}xy = -3$
See 7d) for sketch.

10. a) $9x^2 + 4y^2 = 36$
b) $-9x^2 + y^2 = 9$
c) $x^2 + 4y^2 = 16$

12. a) ellipse
b) $9x^2 + 25y^2 = 225$
c)



13. a) hyperbola
b) $2x^2 - 2y^2 = -1$
c)



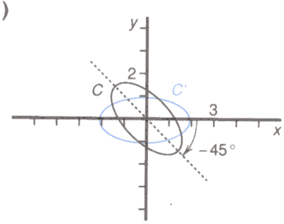
14. Since $f \neq 0$ and $g \neq 0$, the equation cannot be written $ax^2 + 2hxy + by^2 = k$

8.6 Exercises, page 378

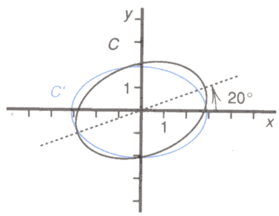
1. a) ellipse
b) 67.5°

2. a) hyperbola, 13.3°
b) ellipse, 45°
c) ellipse, −38.0°
d) hyperbola, −41.8°
e) hyperbola, 39.2°
f) ellipse, 35.8°

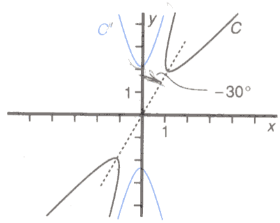
3. a) ellipse
b) 45°
c) $x^2 + 4y^2 = 4$
d)



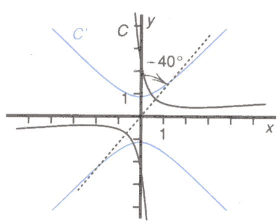
4. a) ellipse, −20°
$4x^2 + 9y^2 = 36$



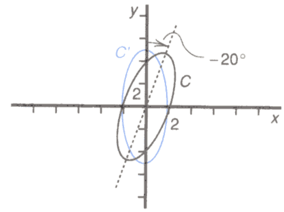
b) hyperbola, 30.0°
$16x^2 - y^2 = -5$



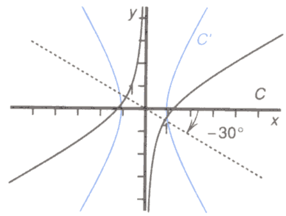
c) hyperbola, 40.1°
$x^2 - y^2 = -1$



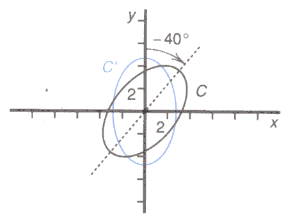
d) ellipse, 20.1°
$25x^2 + 4y^2 = 100$



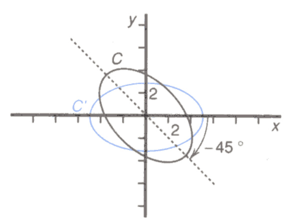
e) hyperbola, 30.0°
$3x^2 - y^2 = 3$



f) ellipse, 40.0°
$25x^2 + 9y^2 = 200$

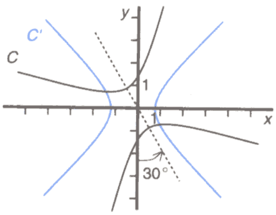


5. a) ellipse
b) 45°
c) $9x^2 + 25y^2 = 225$
d)

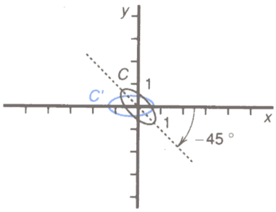


8.6 Exercises, page 378, continued

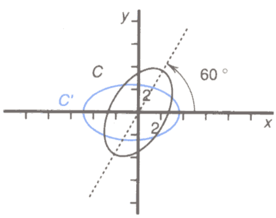
6. **a)** hyperbola 30°
$x^2 - y^2 = 1$



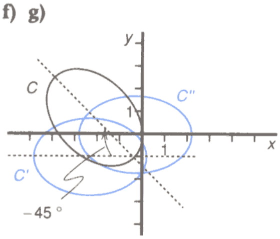
b) ellipse, 45°
$4x^2 + 9y^2 = 4$



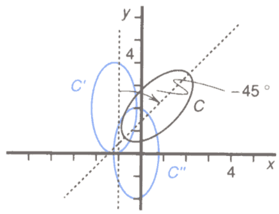
c) ellipse, −60°
$x^2 + 3y^2 = 18$



8. **a)** ellipse
b) 45°
c) $4x^2 + 9y^2 + 16x + 18y = 0$
d) $(x,y) \rightarrow (x + 2, y + 1)$
$4x^2 + 9y^2 = 25$
e) f) g)



9. **a)** ellipse
b) 45°
c) $4x^2 + y^2 + 8x - 4y + 4 = 0$
d) $(x,y) \rightarrow (x + 1, y - 2)$
$4x^2 + y^2 = 4$
e) f) g)



10. same answers as 9

11.
```
100  REM PROGRAM TO
ELIMINATE XY
TERM FROM AX∧2+2XY+BY∧2=K
110  PRINT "STATE THE VALUE
OF A"
120  INPUT A
130  PRINT "STATE VALUE OF
H"
140  INPUT H
150  PRINT "STATE VALUE OF
B"
160  INPUT B
170  PRINT "STATE VALUE OF
K"
180  INPUT K
184  IF A < > B THEN GOTO 190
185  IF A=B GOTO 186
186  T=3.1415925/4
188  GOTO 200
190  T=0.5* ATN
(2*H/(B-A))
200  PRINT "ANGLE OF
ROTATION IS
"T*180/3.14158265"
DEGREES"
210  A1=(A+B)/2+(A-B)/2*
COS(2*T)-H* SIN(2*T)
220  B1=(A+B)/2-(A-B)/2*
COS(2*T)+H* SIN(2*T)
230  H1=(A-B)/2* SIN(2*T)
+H* COS(2*T)
240  PRINT "H="H1
250  PRINT "AN EQUATION OF
THE CONIC IN STANDARD
POSITION IS"
260  PRINT
A1"X∧2+"B1"Y∧2="K
270  STOP
```

Inventory, page 381

1. circle; centre; radius

2. ellipse; centre; ±2; ±3

3. hyperbola; centre; ±2; not real

4. hyperbola; centre; not real; ±3

5. parabola; vertex; (positive) y-axis

6. parabola; vertex; (positive) x-axis

7. translation; square; $6x$; $9x^2$

8. 3; 9

9. $(x - 2, y - 3)$

10. $ab - h^2$; greater than; $ab - h^2$; less than

11. rotation; xy

12. 2θ; $\dfrac{2h}{b - a}$; not equal to; equals; 135°

13. $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$; $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

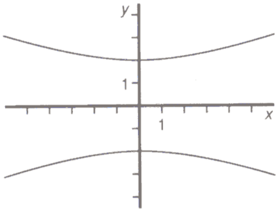
14. 4; 5; 3; $\begin{bmatrix} x \\ y \end{bmatrix}$; $\begin{bmatrix} 4 & 3 \\ 3 & 5 \end{bmatrix}$; [7]

15. $\begin{bmatrix} \cos 20° & -\sin 20° \\ \sin 20° & \cos 20° \end{bmatrix}$, $\begin{bmatrix} 5 & 3 \\ 3 & 7 \end{bmatrix}$

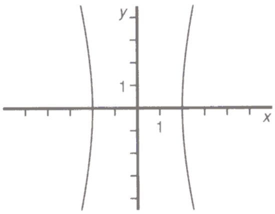
Review Exercises, pages 382–383

1. a) circle centre (0,0) radius 2

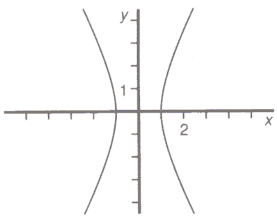
b)



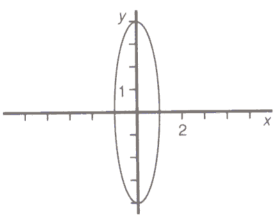
c)



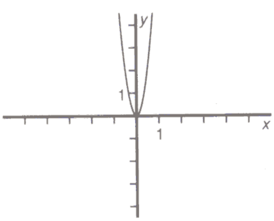
d)



e)

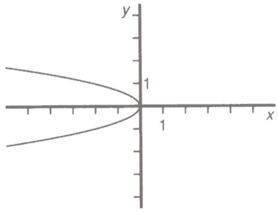


f)



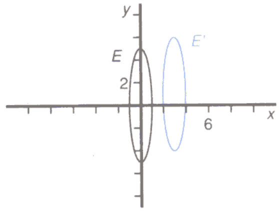
g) circle centre (0,0) radius $\frac{5}{3}$

h)

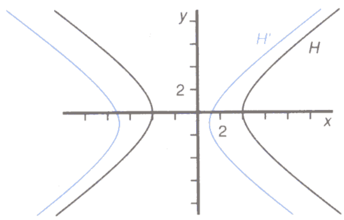


2. a) $25x^2 + y^2 - 150x - 2y + 201 = 0$

b)

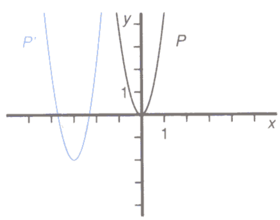


3. a) $9x^2 - 16y^2 + 54x - 32y - 79 = 0$



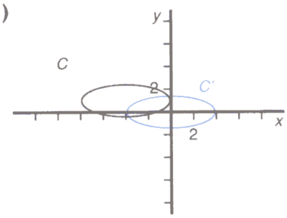
b) $x^2 + y^2 + 4x - 6y - 3 = 0$
C: circle centre (0,0) radius 4
C': circle centre (−2,3) radius 4

c) $4x^2 + 24x - y + 34 = 0$



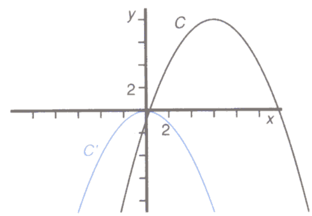
4. a) hyperbola
b) circle
c) parabola
d) ellipse
e) hyperbola

5. a) ellipse
b) $(x,y) \rightarrow (x + 4, y - 1)$
c) $x^2 + 8y^2 = 15$
d)

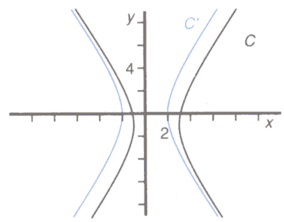


6. a) circle
$(x,y) \rightarrow (x + 3, y + 6)$
$x^2 + y^2 = 47$
C: circle centre (−3,−6)
radius $\sqrt{47}$
C': circle centre (0,0)
radius $\sqrt{47}$

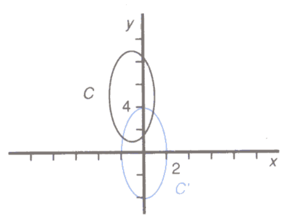
b) parabola
$(x,y) \rightarrow (x - 6, y - 8)$
$x^2 + 4y = 0$



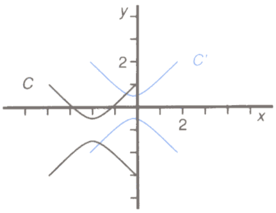
c) hyperbola
$(x,y) \rightarrow (x - 1, y + 1)$
$9x^2 - 4y^2 = 36$



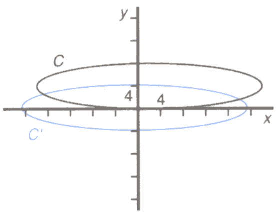
d) ellipse
$(x,y) \rightarrow (x + 1, y - 5)$
$4x^2 + y^2 = 16$



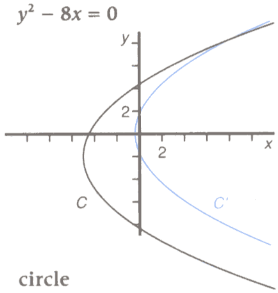
e) hyperbola
$(x,y) \rightarrow (x+2, y+1)$
$4x^2 - 4y^2 = -1$



f) ellipse
$(x,y) \rightarrow (x-2, y-4)$
$x^2 + 25y^2 = 400$



g) parabola
$(x,y) \rightarrow (x+5, y+2)$
$y^2 - 8x = 0$



h) circle
$(x,y) \rightarrow (x-4, y+3)$
$x^2 + y^2 = 26$
C: circle centre $(4,-3)$
radius $\sqrt{26}$
C': circle centre $(0,0)$
radius $\sqrt{26}$

7. **a)** $M = \begin{bmatrix} 5 & -3 \\ -3 & 4 \end{bmatrix}$, $K = [7]$

b) $M = \begin{bmatrix} 3 & -3 \\ -3 & -1 \end{bmatrix}$, $K = [1]$

c) $M = \begin{bmatrix} 2 & 1.5 \\ 1.5 & -1 \end{bmatrix}$, $K = [9]$

d) $M = \begin{bmatrix} 3 & 1 \\ 1 & -5 \end{bmatrix}$, $K = [8]$

e) $M = \begin{bmatrix} 4 & 0 \\ 0 & -12 \end{bmatrix}$, $K = [15]$

f) $M = \begin{bmatrix} 5 & -2 \\ -2 & 5 \end{bmatrix}$, $K = [13]$

8. $[x \quad y]\, M \begin{bmatrix} x \\ y \end{bmatrix} = K$,
using M, K from question 7.

9. **a)** $4x^2 - 6xy + 9y^2 = k$
b) $4x^2 - y^2 = k$
c) $-7x^2 - 8xy + 3y^2 = k$
d) $17x^2 + 20xy + 7y^2 = k$

10. **a)** $\begin{bmatrix} 4 & -3 \\ 5 & 9 \end{bmatrix}$

b) $\begin{bmatrix} -4 \\ 3 \end{bmatrix}$

11. **a)** $\begin{bmatrix} 2 & 0 \\ x & y \end{bmatrix}$

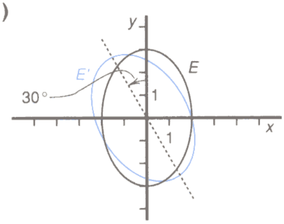
b) $x = 0, y \in \mathbb{R}$

12. **a)** $[x \quad y]\begin{bmatrix} 9 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = [36]$

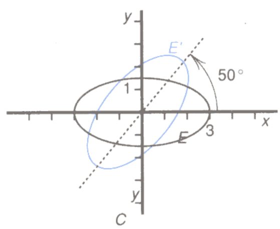
b) $\begin{bmatrix} 7.8 & 2.2 \\ 2.2 & 5.3 \end{bmatrix}$

c) $7.8x^2 + 4.4xy + 5.3y^2 = 36$

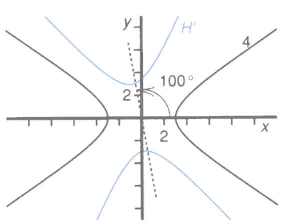
d)



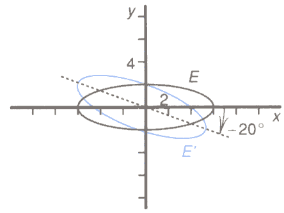
13. **a)** $2.8x^2 - 3.0xy + 2.2y^2 = 9$



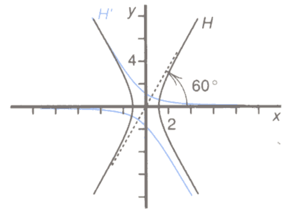
b) $-8.6x^2 - 4.4xy + 3.6y^2 = 36$



c) $1.9x^2 - 5.1xy - 8.0y^2 = 36$



d) $3.5xy + 2y^2 = 4$



14. **a)** $[x \quad y]\begin{bmatrix} 9 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = [36]$

$\begin{bmatrix} \frac{31}{4} & \frac{5\sqrt{3}}{4} \\ \frac{5\sqrt{3}}{4} & \frac{21}{4} \end{bmatrix}$

$\frac{31}{4}x^2 + \frac{5\sqrt{3}}{2}xy + \frac{21}{4}y^2 = 36$

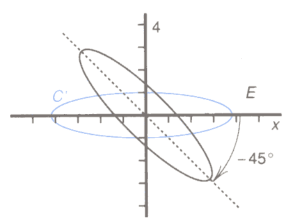
b) See sketch for 12d)

15. **a)** The circle maps to itself.
b) no

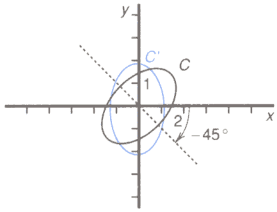
17. **a)** ellipse
b) 13.3°

18. **a)** hyperbola, −7.0°
b) ellipse, 22.5°
c) ellipse, −22.5°
d) ellipse, 45°
e) hyperbola, −41.4°
f) ellipse, 18.4°

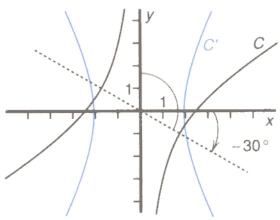
19. **a)** ellipse
b) 45°
c) $x^2 + 16y^2 = 16$
d)



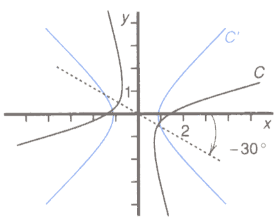
20. a) ellipse, 45°
$3x^2 + y^2 = 4$



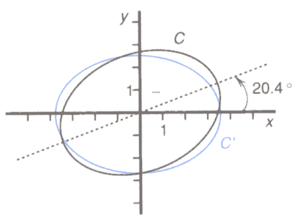
b) hyperbola, 30.0°
$149.9x^2 - 49.9y^2 = 600$



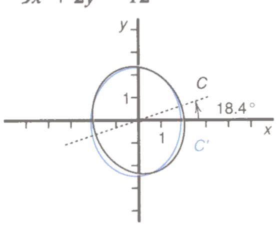
c) hyperbola, 29.9°
$39.9x^2 - 39.9y^2 = 40$



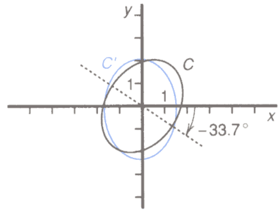
d) ellipse, −20.4°
$26.8x^2 + 53.2y^2 = 360$



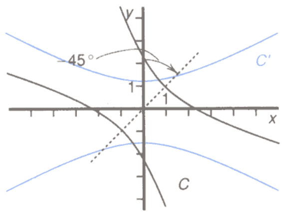
e) ellipse, −18.4°
$3x^2 + 2y^2 = 12$



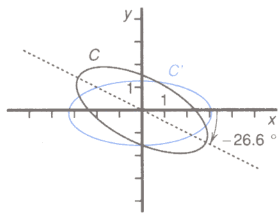
f) ellipse, 33.7°
$2x^2 + y^2 = 5$



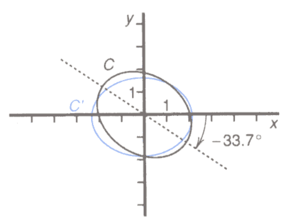
g) hyperbola, 45°
$-4.0x^2 + 14y^2 = 26$



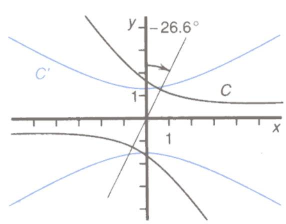
h) ellipse, 26.6°
$x^2 + 5y^2 = 10$



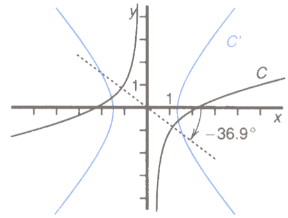
i) ellipse, 33.7°
$3x^2 + 5y^2 = 15$



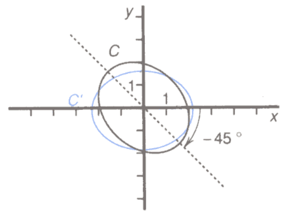
j) hyperbola, 26.6°
$x^2 - 3y^2 = -6$



k) hyperbola, 36.9°
$3x^2 - 2y^2 = 6$

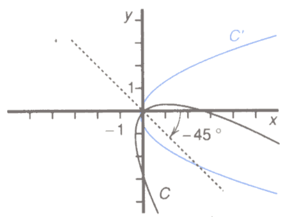


l) ellipse, 45°
$3x^2 + 5y^2 = 15$



21. a) See answer 19.
b) See answer 4b, 8.6 Exercises.

22. a) 45°
b) $y^2 = 2x$
c) parabola
d)



Chapter Nine
Mathematical Induction

9.1 Exercises pages 390–391

1. $\frac{1}{2^n}$
2. a) $2^{n+1} - 2$
   b) $2^n - 1$
3. a) 1, 3, 6, 10
   b) 2, 3, 4, 5
   c) $\frac{n(n+1)}{2}$
4. $\frac{n}{2n+1}$
5. a) $\frac{n}{4n+1}$
   b) $\frac{n}{2n+4}$
   c) $1 - \frac{1}{2^n}$
6. a) 3
   b) 4
   c) 5
   d) 6
   e) $n + 1$
7. a) $\frac{n+2}{2(n+1)}$
   b) $(n+1)^2$
8. a) 2, 8, 20, 40
   b) all $n \in \mathbb{N}$
9. all $n \in \mathbb{N}$
10. a) all
    b) all
    c) all
    d) all
    e) even $n \in \mathbb{W}$
    f) even $n \in \mathbb{W}$
11. $n \geqslant 4$
12. a) $n \geqslant 7$
    b) $n \geqslant 2$
    c) all n
13. a) 1, 9, 36, 100, 225, 441, 784
    b) 1, 3, 6, 10, 15, 21, 28
    c) $\left[\frac{n(n+1)}{2}\right]^2$
14. a) 3
    b) 6
    c) 10
    d) $\frac{n(n-1)}{2}$
15. a) 2
    b) 5
    c) 9, 14
    d) $\frac{n(n-3)}{2}$
16. a) 4
    b) 7
    c) 11
    d) $\frac{n^2+n+2}{2}$

9.2 Exercises, page 395

1. *Step 1* Show the statement is true for $n = 1$.
   Step 2 Assume the statement is true for $n = k$.
   Step 3 Prove the statement is true for $n = k + 1$, using result of step 2.
5. a) 43, 47, 53, 61
   b) $41^2 + 41 + 41 = 41(41 + 1 + 1) = 41 \times 43$
   c) steps 2 and 3
6. a) step 1
   b) no

9.3 Exercises, page 398

No answers provided.

9.4 Exercises, page 405

1. a)

1	1							
1	2	1						
1	3	3	1					
1	4	6	4	1				
1	5	10	10	5	1			
1	6	15	20	15	6	1		
1	7	21	35	35	21	7	1	
1	8	28	56	70	56	28	8	1

   b) 24, 720
   c) 35
   d) 1, 5, 10, 10, 5, 1
2. a) $C(4,0)a^4x^0 + C(4,1)a^3x^1 + C(4,2)a^2x^2 + C(4,3)a^1x^3 + C(4,4)a^0x^4$
   b) $C(5,0)a^5x^0 + C(5,1)a^4x^1 + C(5,2)a^3x^2 + C(5,3)a^2x^3 + C(5,4)a^1x^4 + C(5,5)a^0x^5 +$
   c) $C(6,0)a^6x^0 + C(6,1)a^5x^1 + C(6,2)a^4x^2 + C(6,3)a^3x^3 + C(6,4)a^2x^4 + C(6,5)a^1x^5 + C(6,6)a^0x^6$
   d) $C(7,0)a^7x^0 + C(7,1)a^6x^1 + C(7,2)a^5x^2 + C(7,3)a^4x^3 + C(7,4)a^3x^4 + C(7,5)a^2x^5 + C(7,6)a^1x^6 + C(7,7)a^0x^7$
   e) $C(8,0)a^8x^0 + C(8,1)a^7x^1 + C(8,2)a^6x^2 + C(8,3)a^5x^3 + C(8,4)a^4x^4 + C(8,5)a^3x^5 + C(8,6)a^2x^6 + C(8,7)a^1x^7 + C(8,8)a^0x^8$
   f) $C(9,0)a^9x^0 + C(9,1)a^8x^1 + C(9,2)a^7x^2 + C(9,3)a^6x^3 + C(9,4)a^5x^4 + C(9,5)a^4x^5 + C(9,6)a^3x^6 + C(9,7)a^2x^7 + C(9,8)a^1x^8 + C(9,9)a^0x^9$
3. a) $a^4 + 4a^3x + 6a^2x^2 + 4ax^3 + x^4$
   b) $a^5 + 5a^4x + 10a^3x^2 + 10a^2x^3 + 5ax^4 + x^5$
   c) $a^6 + 6a^5x + 15a^4x^2 + 20a^3x^3 + 15a^2x^4 + 6ax^5 + x^6$
   d) $a^7 + 7a^6x + 21a^5x^2 + 35a^4x^3 + 35a^3x^4 + 21a^2x^5 + 7ax^6 + x^7$
   e) $a^8 + 8a^7x + 28a^6x^2 + 56a^5x^3 + 70a^4x^4 + 56a^3x^5 + 28a^2x^6 + 8ax^7 + x^8$
   f) $a^9 + 9a^8x + 36a^7x^2 + 84a^6x^3 + 126a^5x^4 + 126a^4x^5 + 84a^3x^6 + 36a^2x^7 + 9ax^8 + x^9$
4. a) $a^4 + 4a^3y + 6a^2y^2 + 4ay^3 + y^4$
   b) $b^4 - 4b^3c + 6b^2c^2 - 4bc^3 + c^4$
   c) $m^3 + 3m^2z + 3mz^2 + z^3$
   d) $32 + 80x + 80x^2 + 40x^3 + 10x^4 + x^5$
   e) $a^8 + 8a^7 + 28a^6 + 56a^5 + 70a^4 + 56a^3 + 28a^2 + 8a + 1$
   f) $81 - 108b + 54b^2 - 12b^3 + b^4$
5. a) $a^4 + 8a^3b + 24a^2b^2 + 32ab^3 + 16b^4$
   b) $81a^4 + 432a^3b + 864a^2b^2 + 768ab^3 + 256b^4$
   c) $27 - 54m + 36m^2 - 8m^3$
   d) $64a^3 - 240a^2 + 300a - 125$
   e) $32x^5 + 240x^4a + 720x^3a^2 + 1080x^2a^3 + 810xa^4 + 243a^5$
   f) $1 - 6m^2 + 15m^4 - 20m^6 + 15m^8 - 6m^{10} + m^{12}$
6. a) $C(40,0)a^{40}b^0 + C(40,1)a^{39}b^1 + C(40,2)a^{38}b^2 + C(40,3)a^{37}b^3$
   b) $C(34,0)m^{34}(-k)^0 + C(34,1)m^{33}(-k)^1 + C(34,2)m^{32}(-k)^2 + C(34,3)m^{31}(-k)^3$
   c) $C(23,0)3^{23}x^0 + C(23,1)3^{22}x^1 + C(23,2)3^{21}x^2 + C(23,3)3^{20}x^3$

d) $C(85,0)4^{85}(2a)^0 +$
$C(85,1)4^{84}(2a)^1 +$
$C(85,2)4^{83}(2a)^2 +$
$C(85,3)4^{82}(2a)^3$

e) $C(25,0)(2m)^{25}(-3t)^0 +$
$C(25,1)(2m)^{24}(-3t)^1 +$
$C(25,2)(2m)^{23}(-3t)^2 +$
$C(25,3)(2m)^{22}(-3t)^3$

f) $C(36,0)1^{36}(b^2)^0 +$
$C(36,1)1^{35}(b^2)^1 +$
$C(36,2)1^{34}(b^2)^2 +$
$C(36,3)1^{33}(b^2)^3$

7. a) $x^4 + 4x^2 + 6 + \frac{4}{x^2} + \frac{1}{x^4}$

b) $x^5 - 10x^2 + \frac{40}{x} - \frac{80}{x^4} + \frac{80}{x^7} - \frac{32}{x^{10}}$

8. a) $C(6,k)x^{12-3k}$
b) $6x^9$
c) 15

11. 4

14. a) $1 - 2x + 3x^2 - 4x^3$

b) $1 + \frac{x}{2} - \frac{x^3}{8} + \frac{x^4}{4}$

Inventory, page 409

1. 1
2. k; in step 3 prove; $k + 1$
3. a) sometimes
b) sometimes
4. a) 1; 1
b) $1 + 3 + 5 + \ldots + (2k - 1)$; k^2
c) $1 + 3 + 5 + \ldots + (2k + 1)$; $(k + 1)^2$
5. a) 2; 2
b) $(1 + 1)\left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{3}\right)\ldots\left(1 + \frac{1}{k}\right)$; $k + 1$
c) $(1 + 1)\left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{3}\right)\ldots\left(1 + \frac{1}{k+1}\right)$; $k + 2$
6. a) $f(1) = \frac{6}{3} = 2$
is a natural number
b) $f(k) = \frac{k^3 + 3k^2 + 2k}{3}$
is a natural number
c) $f(k + 1) = \frac{(k + 1)^3 + 3(k + 1)^2 + 2(k + 1)}{3}$
is a natural number
7. a) $2^4 < 4!$
b) $2^k < k!$
c) $2^{k+1} < (k + 1)!$
8. 6;
$C(5,0)a^5x^0 + C(5,1)a^4x^1 +$
$C(5,2)a^3x^2 + C(5,3)a^2x^3 +$
$C(5,4)a^1x^4 + C(5,5)a^0x^5$
9. 1, 9, 36, 84, 126, 84, 36, 9, 1

Review Exercises, pages 410–411

1. *Step 1* Show the statement is true for $n = 1$.
Step 2 Assume the statement is true for $n = k$.
Step 3 Prove the statement is true for $n = k + 1$, using result of step 2.

3. n^3

10. step 1

13. b) not true for $n = 1$

15. a) $a^4 + 4a^3x + 6a^2x^2 + 4ax^3 + x^4$
b) $243 + 405b + 270b^2 + 90b^3 + 15b^4 + b^5$
c) $8 + 12x^2 + 6x^2 + x^3$
d) $64k^6 - 960k^5m + 6000k^4m^2 - 20\,000k^3m^3 + 37\,500k^2m^4 - 37\,500km^5 + 15\,625m^6$

16. not true for $n = 4$

21. 7

23. i) $\frac{1}{2}n(n + 1)$

iii) $\frac{1}{6}(n + 1)(n + 2)(2n + 3) - 5$

Chapter Ten
Complex Numbers

10.1 Exercises, page 418

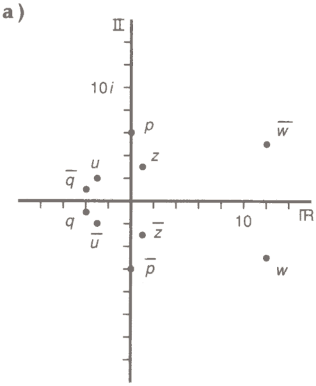
1. a) $9 + 3i$
   b) 1
   c) $17 + 7i$
   d) 10
   e) $-3 + 4i$
   f) $-3 - 4i$
   g) $-4 + 6i$
   h) $-11 - 2i$
2. $\dfrac{7 \pm 5i}{2}$
3. $2 \pm i$
4. $-\dfrac{1}{2} \pm i\dfrac{\sqrt{7}}{2}$
5. a) $4 \pm 3i$
   b) $-2 \pm i$
   c) $\dfrac{5 \pm i\sqrt{59}}{6}$
7. $(4 + 3i) + (4 - 3i) = 8$
   $(4 + 3i)(4 - 3i) = 25$
   order does not matter
8. sum $-\dfrac{b}{a}$; product $\dfrac{c}{a}$
9. a) 0 or $4i$
   b) $4i$ or $-i$
   c) $i\left(\dfrac{1 \pm \sqrt{13}}{2}\right)$
   d) $2i$ or $1 - i$
10. a) 4,6
    b) 0,7
    c) 9,7
    d) 26,0
    e) 0,2
    f) 7,−24
11. $z + w = (a + c) + i(b + d)$
    $= w + z$
    $\Rightarrow$ commutative
12. $(z + w) + u = (a + c + e)$
    $+ i(b + d + f)$
    $= z + (w + u)$
    $\Rightarrow$ associative
13. $zw = (ac - bd) + i(ad + bc)$
    $= wz$
    $\Rightarrow$ commutative
14. $(zw)u = (ace - bde - adf - bcf)$
    $+ i(ade + bce + acf - bdf)$
    $= z(wu)$
    $\Rightarrow$ associative
15. $zw + zu = (ac - bd + ae - bf)$
    $+ i(ad - bc + af - be)$
    $= z(w + u)$
    $\Rightarrow$ distributive over addition

10.2 Exercises, page 422

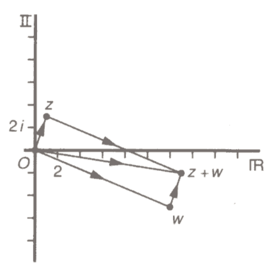
1. a) $-i$
   b) 1
   c) i
   d) -1
   e) $-i$
   f) -1
   g) 1
   h) i
   j) -1
2. a) $1 - 4i$
   b) $-2 + 2i$
   c) $6i$
   d) $-2 + 16i$
   e) $52 - 8i$
3. a) $4 - i$
   b) $\dfrac{6}{5} + \dfrac{7}{5}i$
   c) $3 + 7i$
   d) $\dfrac{21}{29} + \dfrac{20}{29}i$
   e) $\dfrac{2}{13} - \dfrac{3}{13}i$
   f) $\dfrac{4}{25} + \dfrac{3}{25}i$
   g) $\dfrac{6}{25}$
   h) $\dfrac{355}{3721} - \dfrac{245}{3721}i$
5. c) Yes.
8. b) $\dfrac{42}{13} - \dfrac{24}{13}i$
9. $-\dfrac{b}{2a} \pm i\dfrac{\sqrt{4ac - b^2}}{2a}$
   $\Rightarrow$ conjugates
10. a) $x = 17, y = 0$
    b) $x = \dfrac{4}{17}, y = -\dfrac{18}{17}$
11. a) $\mathrm{Re}(z) = \dfrac{7}{13}$
    $\mathrm{Im}(z) = 1$
    b) $\mathrm{Re}(z) = -4$
    $\mathrm{Im}(z) = 0$
12. $\mathrm{Re}(z) = 3$, $\mathrm{Im}(z) = 3$
    $\mathrm{Re}(z^2) = 0$, $\mathrm{Im}(z^2) = 18$
13. 10 000
14. $-\dfrac{3\sqrt{5}}{2} + 5i$ or $\dfrac{3\sqrt{5}}{2} + 7i$
15. $\cos\alpha \pm i \sin\alpha$

10.3 Exercises, page 429

1. a)



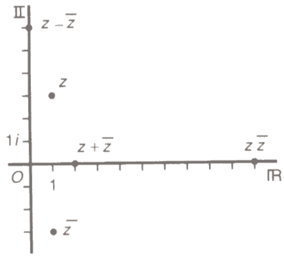
   b) $\overline{z} = 1 - 3i$
   $\overline{w} = 12 + 5i$
   $\overline{p} = -6i$
   $\overline{q} = -4 + i$
   $\overline{u} = -3 - 2i$
2. a) $|z| = \sqrt{10}$
   $|w| = 13$
   $|p| = 6$
   $|q| = \sqrt{17}$
   $|u| = \sqrt{13}$
   b) $\arg z = 72°$
   $\arg w = -23°$
   $\arg p = 90°$
   $\arg q = -166°$
   $\arg u = 146°$
3. a) $|\overline{z}| = \sqrt{10}, |\overline{w}| = 13$
   b) $\arg \overline{z} = -72°$, $\arg \overline{w} = 23°$
   c) same modulus, opposite argument,
4. a) impossible
   b) $|z| < |u| < |q| < |p| < |w|$
5. a) $z + w = 13 - 2i$
   b)



   c) numbers represented by vectors, as shown

6. a) $z + \bar{z} = 2$
$z - \bar{z} = 6i$
$z\bar{z} = 10$

b)

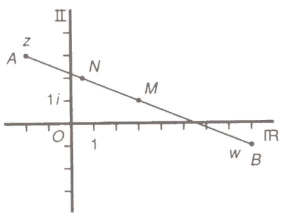


7. a) If $a \in \mathbb{R}$,
$|a| = \sqrt{a^2} = \pm a$
$\arg a = 0°$ or $180°$
$\bar{a} = a$

b) If $b \in \mathbb{I}$,
$|b| = \sqrt{-b^2}$
$\arg b = \pm 90°$
$\bar{b} = -b$

8. $|z| = 1, |\bar{w}| = 3$

9. a)



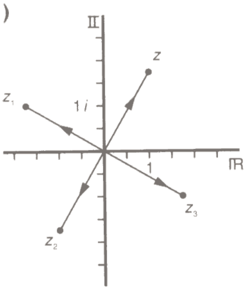
b) $3 + i$

c) $\frac{1}{2} + 2i$

d) M midpoint of AB,
N divides AB in ratio 1 : 3

10. a) $z_1 = -\sqrt{3} + i$
$z_2 = -1 - i\sqrt{3}$
$z_3 = \sqrt{3} - i$

b)

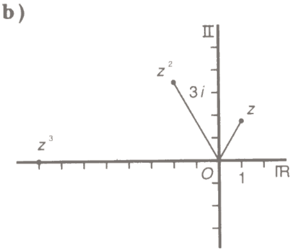


c) $|z| = |z_1| = |z_2| = |z_3| = 2$
$\arg z = 60°$
$\arg z_1 = 150°$
$\arg z_2 = 240°$
$\arg z_3 = 330°$

d) rotates counterclockwise about O, by $90°$

11. a) $z^2 = -2 + 2i\sqrt{3}$
$z^3 = -8$

b)



c) It is true that
$(1 + i\sqrt{3})^3 = -8$, but
$(-2)^3 = -8$ also!

12. a) $|z| = 3\sqrt{2}$
$\arg z = 45°$

10.4 Exercises, page 433

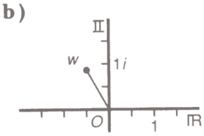
1. a) 2 or −3
b) $1 \pm i$
c) $\frac{1}{4}$ or $\pm i$
d) 0 or $-2i$ or $\frac{3}{2} + 2i$

2. a) b) c)

3. a) $z^2 - (2 + 5i)z - 4 + 8i = 0$
b) $z^2 - 2pz + p^2 + q^2 = 0$

5. a) $z^3 - (3 + 2i)z^2 + (13 + 7i)z - 20(1 + i) = 0$
b) $z^3 - 2pz^2 + (p^2 + q^2)z = 0$

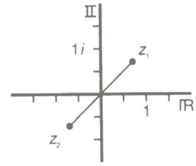
7. a) $w^3 = 1$
b)



8. $x = \frac{7}{34}, y = -\frac{11}{34}$

9. $x = -\frac{(ac + bd)}{a^2 + b^2}, y = \frac{bc - ad}{a^2 + b^2}$;
yes

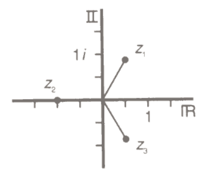
10. $z_1 = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$ or
$z_2 = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$



11. $\frac{3 + 3\sqrt{2}}{2} + i\frac{(1 + 3\sqrt{2})}{2}$ or
$\frac{3 - 3\sqrt{2}}{2} + i\frac{(1 - 3\sqrt{2})}{2}$

12. a) $z_1 = \frac{1}{2} + \frac{\sqrt{3}}{2}i$
$z_2 = -1$
$z_3 = \frac{1}{2} - \frac{\sqrt{3}}{2}i$

b)

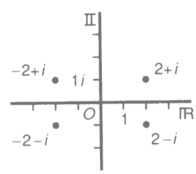


13. a) true in $\mathbb{R}$, but in $\mathbb{C}$, $z = \pm iw$
b) true in $\mathbb{R}$ only;
in $\mathbb{C}$, $z = w$ or
$z = \frac{-w \pm iw\sqrt{3}}{2}$

14. b) $\frac{2}{3}$ or $-\frac{1}{2}$ or $2 \pm i$

15. a) real coefficients, so
root $2 + i \Rightarrow$ root $2 - i$
root $-2 + i \Rightarrow$ root $-2 - i$

b)

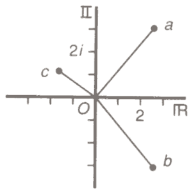


16. $z = r$ is the only real root.

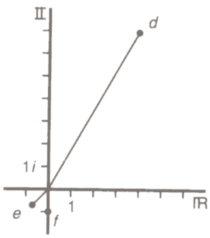
17. a) $r = 4, s = -5$
b) 5

10.5 Exercises, page 439

1. a) $a = 2.57 + 3.06i$
   $b = 2.57 - 3.06i$
   $c = -1.64 + 1.15i$



2. $d = 4 + 4i\sqrt{3}$
   $e = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$
   $f = -i$

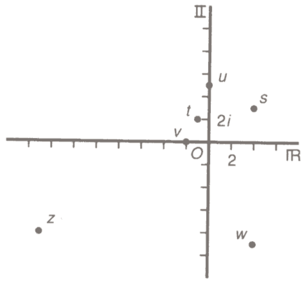


3. a) $|z| = 3, \arg z = \frac{\pi}{2}$
   b) $|z| = 4, \arg z = 0$
   c) $|z| = 17, \arg z = \pi$
   d) $|z| = 1, \arg z = -\frac{\pi}{2}$

4. a) $|\bar{z}| = 3, \arg \bar{z} = -\frac{\pi}{2}$
   b) $|\bar{z}| = 4, \arg \bar{z} = 0$
   c) $|\bar{z}| = 17, \arg \bar{z} = \pi$
   d) $|\bar{z}| = 1, \arg \bar{z} = \frac{\pi}{2}$

5. a) $115°$
   b) $65°$
   c) $-\frac{2\pi}{3}$
   d) $-\frac{\pi}{6}$

6. $s = 5(\cos 37° + i \sin 37°)$
   $t = \sqrt{5}(\cos 117° + i \sin 117°)$
   $u = 5(\cos 90° + i \sin 90°)$
   $v = 2(\cos 180° + i \sin 180°)$
   $z = 17(\cos 152° - i \sin 152°)$
   $w = \sqrt{97}(\cos 66° - i \sin 66°)$



7. a) $\sqrt{2}\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)$
   b) $2\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)$
   c) $4\sqrt{3}\left(\cos \frac{\pi}{3} - i \sin \frac{\pi}{3}\right)$
   d) $2\sqrt{3}\left(\cos \frac{5\pi}{6} - i \sin \frac{5\pi}{6}\right)$

8. 0 or π

9. a) $r = 5, \theta = 30°$
   b) $r = 6, \theta = -32°$
   c) $r = 1, \theta = -\frac{\pi}{8}$

11. a) $50(\cos 105° + i \sin 105°)$
    b) $2(\cos 37° + i \sin 37°)$
    c) $\frac{1}{2}(\cos 37° - i \sin 37°)$

12. a) $z = \sqrt{41}(\cos 51° - i \sin 51°)$
    $w = \sqrt{13}(\cos 124° + i \sin 124°)$
    b) $zw = \sqrt{533}(\cos 73° - i \sin 73°)$
    $\frac{z}{w} = \sqrt{\frac{41}{13}}(\cos 175° - i \sin 175°)$
    $\frac{w}{z} = \sqrt{\frac{13}{41}}(\cos 175° + i \sin 175°)$

13. $|z| = 2, \arg z = \frac{2\pi}{3}$
    $|w| = 4, \arg w = \frac{\pi}{6}$

14. a) $4\left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}\right)$
    b) $16\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$
    c) $8\left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}\right)$
    d) $2\left(\cos \frac{\pi}{2} - i \sin \frac{\pi}{2}\right)$

15. a) $|i| = 1, \arg i = \frac{\pi}{2}$
    b) length unchanged;
    rotates about origin, by $\frac{\pi}{2}$

16. a) $\arg(z^2) = 2\theta$

10.6 Exercises, page 443

1. a) $-0.766 - 0.643i$
   b) $-365 - 631i$
   c) $-928 - 433i$

2. a) 65 536
   b) 4096
   c) -1
   d) $-\frac{1}{2} - i\frac{\sqrt{3}}{2}$

3. a) $-0.766 + 0.643i$
   b) $-0.000\,686 + 0.001\,19i$
   c) $-0.000\,885 + 0.000\,413i$

4. a) $\frac{1}{2^{16}}$
   b) $\frac{1}{2^{12}}$
   c) -1
   d) $-\frac{1}{2} + i\frac{\sqrt{3}}{2}$

5. a) -1024
   b) $-32i$

6. a) -1
   b) 1

7. $\cos 4\theta =$
   $\cos^4 \theta - 6 \cos^2 \theta \sin^2 \theta + \sin^4 \theta$
   $\sin 4\theta =$
   $4 \cos \theta \sin \theta(\cos^2 \theta - \sin^2 \theta)$

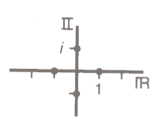
8. d) $\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$

9. b) $\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$

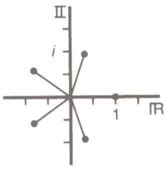
10. c) $\cos 4\theta =$
    $8 \cos^4 \theta - 8 \cos^2 \theta + 1$

10.7 Exercises, page 448

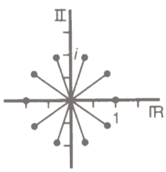
2. a) $w_0 = 1$
   $w_1 = i$
   $w_2 = -1$
   $w_3 = -i$



b) $w_0 = 1$
$w_1 = 0.309 + 0.951i$
$w_2 = -0.809 + 0.588i$
$w_3 = -0.809 - 0.588i$
$w_4 = 0.309 - 0.951i$

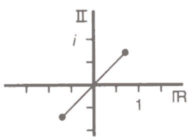


c) $w_0 = 1$
$w_1 = 0.809 + 0.588i$
$w_2 = 0.309 + 0.951i$
$w_3 = -0.309 + 0.951i$
$w_4 = -0.809 + 0.588i$
$w_5 = -1$
$w_6 = -0.809 - 0.588i$
$w_7 = -0.309 - 0.951i$
$w_8 = 0.309 - 0.951i$
$w_9 = 0.809 - 0.588i$



3. a) $w_0 = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$

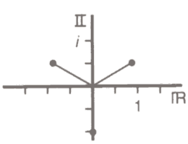
$w_1 = -\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$



b) $w_0 = \frac{\sqrt{3}}{2} + \frac{i}{2}$

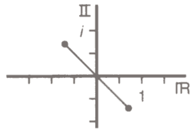
$w_1 = -\frac{\sqrt{3}}{2} + \frac{i}{2}$

$w_2 = -i$

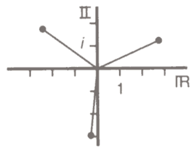


c) $w_0 = -\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$

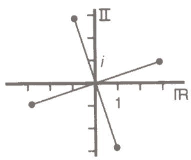
$w_1 = \frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$



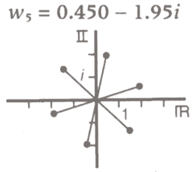
d) $w_0 = 2.74 + 1.22i$
$w_1 = -2.43 + 1.76i$
$w_2 = -0.314 - 2.98i$



e) $w_0 = 2.85 + 0.927i$
$w_1 = -0.927 + 2.85i$
$w_2 = -2.85 - 0.927i$
$w_3 = 0.927 - 2.85i$



f) $w_0 = 1.91 - 0.585i$
$w_1 = 1.46 + 1.36i$
$w_2 = 0.450 + 1.95i$
$w_3 = -1.91 + 0.585i$
$w_4 = -1.46 - 1.36i$
$w_5 = 0.450 - 1.95i$



4. $2\left(\cos\frac{2\pi}{5} - i\sin\frac{2\pi}{5}\right)$ or

$2\left(\cos\frac{4\pi}{5} - i\sin\frac{4\pi}{5}\right)$ or 2

5. a) $z \in$
$\left\{1, \cos\frac{2\pi}{5} \pm i\sin\frac{2\pi}{5}, \cos\frac{4\pi}{5} \pm i\sin\frac{4\pi}{5}\right\}$

b) $z^5 - 1 =$

$$(z-1)\left(z - \left[\cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}\right]\right)$$
$$\left(z - \left[\cos\frac{2\pi}{5} - i\sin\frac{2\pi}{5}\right]\right)$$
$$\left(z - \left[\cos\frac{4\pi}{5} + i\sin\frac{4\pi}{5}\right]\right)$$
$$\left(z - \left[\cos\frac{4\pi}{5} - i\sin\frac{4\pi}{5}\right]\right)$$

c) $z^5 - 1 =$

$$(z-1)\left(z^2 - 2\cos\frac{2\pi}{5}z + 1\right)$$
$$\left(z^2 - 2\cos\frac{4\pi}{5}z + 1\right)$$

6. a) $(z+1)$
$\left(z^2 - 2\cos\frac{\pi}{5}z + 1\right)$
$\left(z^2 - 2\cos\frac{3\pi}{5}z + 1\right)$

b) $(z-1)$
$\left(z^2 - 2\cos\frac{2\pi}{7}z + 1\right)$
$\left(z^2 - 2\cos\frac{4\pi}{7}z + 1\right)$
$\left(z^2 - 2\cos\frac{6\pi}{7}z + 1\right)$

c) $(z+1)$
$(z-1)$
$\left(z^2 - 2\cos\frac{\pi}{3}z + 1\right)$
$\left(z^2 - 2\cos\frac{2\pi}{3}z + 1\right)$

7. b) other roots are
$\sqrt{3} + i, -\sqrt{3} + i, -\sqrt{3} - i$

8. a) The arguments of the non-real roots of unity are the multiples of $\frac{2\pi}{7}$, with coefficients in $S = \{1,2,3,4,5,6\}$
Taking any one of these, and multiplying by the numbers 1-6 yields results as follows. (This is known as multiplication modulo 7.)

x	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

Each row yields all the members of S.

10.7 Exercises, page 448, continued

b) Let
$1 + w + w^2 + w^3 + w^4 + w^5 + w^6$
$= z,$
If w is a non-real 7th root,
$w + w^2 + w^3 + w^4 + w^5 + w^6 + w^7$
$= zw,$
or
$w + w^2 + w^3 + w^4 + w^5 + w^6 + 1$
$= zw$
$\Rightarrow z = zw$
$\Rightarrow z(1 - w) = 0$
$\Rightarrow z = 0$
(since $w \neq 1$)

c) No: a multiplication table as above holds only if n is prime.

9. a) $\dfrac{w_{k+1}}{w_k} = \cos 72^\circ + i \sin 72^\circ$

b) Each root is the previous root, rotated by 72°.

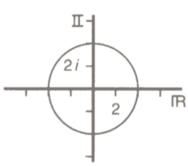
10. a) $(z + 1)$
$$\left(z^2 - 2\cos\frac{\pi}{7}z + 1\right)$$
$$\left(z^2 - 2\cos\frac{3\pi}{7}z + 1\right)$$
$$\left(z^2 - 2\cos\frac{5\pi}{7}z + 1\right) = 0$$

b) On expansion all powers of z have zero coefficients, except z^7; the coefficient of z is
$$\left(1 - 2\cos\frac{\pi}{7} - 2\cos\frac{3\pi}{7} - 2\cos\frac{5\pi}{7}\right),$$
which equals zero.
Thus,
$$\cos\frac{\pi}{7} + \cos\frac{3\pi}{7} + \cos\frac{5\pi}{7} = \frac{1}{2}$$

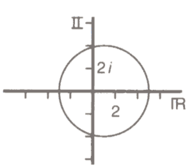
11. $z = \sqrt[3]{2}(\cos k\pi \pm i \sin k\pi)$,
$k = \frac{1}{9}, \frac{5}{9}, \frac{7}{9}$

10.8 Exercises, page 453

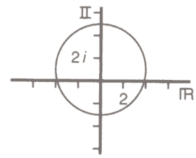
1. a)



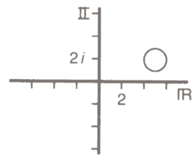
b)



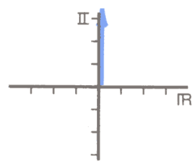
c)



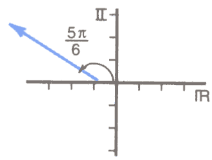
d)



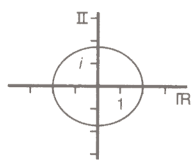
e)



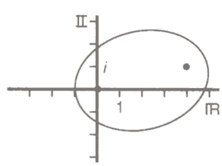
f)



g)



h)



2. a) $|z| = 6$
b) $|z + 1 - 3i| = 5$
c) $|z - u| = a$

3. a) $(x + 4)^2 + (y + 3)^2 = 4$
b) $x^2 + \left(y + \frac{5}{4}\right)^2 = \frac{9}{16}$

5. $z = -(2 + i) + pi$

6. $x^2 + y^2 = 1$ or $x = 0$

7. $y = -\sqrt{3}x + 2 + 4\sqrt{3}$ and $x < 4$

8. $x = 0$ or $y = 0$

9. $y = 0$ or $x^2 + y^2 = 2$

10. a) $|z - 2| = |z + 6|$
b) $|z - 2 - i| = |z - 3 + 2i|$

12. a) $x = \frac{9}{2}$
b) $\left(x - \frac{8}{3}\right)^2 + \left(y - \frac{4}{3}\right)^2 = \frac{80}{9}$
c) $(x + 1)^2 + (y - 1)^2 = 2$
d) $\left(x + \frac{1}{2}\right)^2 + (y - 1)^2 = \frac{9}{4}$

13. $xy = 1$

14. a) interior of circle: centre O, radius 5
b) interior of circle and circumference: centre $5 - 3i$, radius 3
c) 'exterior' of hyperbola $x^2 - y^2 = 2$
d) annulus: centre $2i$, radii 2 and 3

15. a) parabola
b) $y^2 = 4x$

16. a) circle: centre $2 + 3i$ radius 4
b) line segment

17. a) $|z| \doteq 7.61$
b) $|z| = 4$

10.9 Exercises, page 461

1. a) $r^2e^{2i\theta}$
b) $r^3e^{3i\theta}$
c) $\frac{1}{r}e^{-i\theta}$
d) $re^{-i\theta}$

2. a) $z = 2.24e^{0.46i}$
b) $w = 3.16e^{-1.89i}$

3. a) $u = 10e^{-\frac{i\pi}{3}}$
b) $v = 3\sqrt{2}e^{\frac{3i\pi}{4}}$

4. a) $4e^{\frac{4i\pi}{5}}$
b) 32
c) $\frac{1}{2}e^{-\frac{2i\pi}{5}}$
d) $\frac{1}{4}e^{-\frac{4i\pi}{5}}$
e) $\sqrt{2}e^{\frac{i\pi}{5}}$
f) $-4\sqrt{2}$

6. $e^{2ki\pi} = 1, k \in \mathbb{Z}$
An infinite number of different arguments give the same number.

10. a) $k\pi$

b) $\left(k+\dfrac{1}{2}\right)\pi$

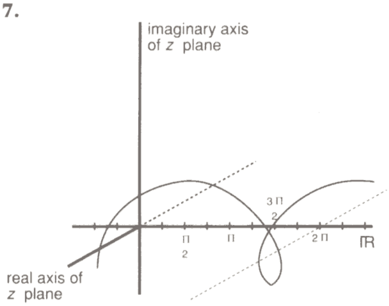
11. $z^{\frac{1}{2}} = r^{\frac{1}{2}} e^{\frac{i\theta}{2}} = w_0$, or
$z^{\frac{1}{2}} = -r^{\frac{1}{2}} e^{\frac{i\theta}{2}} = w_1$

16. $f'(\theta) = ie^{i\theta}$
$= i(\cos\theta + i\sin\theta)$
$= i\cos\theta - \sin\theta$

or

$f'(\theta) = -\sin\theta + i\cos\theta$

17.



Inventory, page 466

1. -1
2. imaginary
3. $a + ib$ or $a + bi$
4. $\mathbb{R} \subset \mathbb{C}$
5. real; imaginary
6. non-real
7. $a = c$; $b = d$
8. a; b; $a - bi$;
$\sqrt{a^2+b^2}$;
$\tan(\arg z) = \dfrac{b}{a}$
9. reflections;
real axis
10. z
11. Complex
12. $r(\cos\theta + i\sin\theta)$
13. equal;
any multiple of 2Π
14. product
15. difference
16. $r^n(\cos n\theta + i\sin n\theta)$
17. $z^5 - 1 = 0$
18. n
19. two
20. $|z-w|$ or $|w-z|$
21. e^x; y
22. -1

Review Exercises, pages 467–471

1. a) 10
b) $25 - 8i$
c) $120 - 22i$
d) $-2 + 2i$
e) $-i$
f) 1
g) i
h) $-2 + 6i$
i) $-7 + i$
j) $24i$
k) $-6 - 11i$

2. a) $-3 - i$
b) $-\dfrac{4}{5} - \dfrac{3}{5}i$
c) $-5 + 8i$
d) $\dfrac{9}{10} + \dfrac{7}{10}i$
e) $\dfrac{3}{25} + \dfrac{4}{25}i$
f) $\dfrac{1}{5} + \dfrac{2}{5}i$
g) $\dfrac{267}{962} + \dfrac{24}{481}i$
h) $-\dfrac{688}{7225} - \dfrac{134}{7225}i$

3. a) $4bi$
b) $\dfrac{2b}{a^2+b^2}i$
c) $\dfrac{a(a^2+b^2+1)}{a^2+b^2} + \dfrac{ib(a^2+b^2-1)}{a^2+b^2}$

4. $5 \pm 2i$

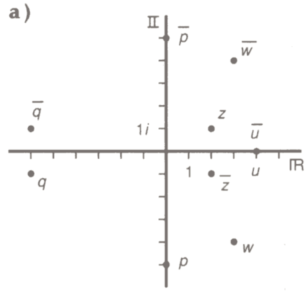
5. a) $6 \pm i$
b) $-2 \pm 4i$
c) $\dfrac{3 \pm i\sqrt{11}}{2}$

6. 4 or i

8. 3 570 125

9. $k = \pm\sqrt{46}$

10. a)



b) $\bar{z} = 2 - i$
$\bar{w} = 3 + 4i$
$\bar{p} = 5i$
$\bar{q} = -6 + i$
$\bar{u} = 4$

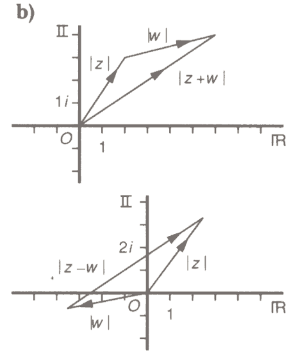
c) $|z| = \sqrt{5}$
$|w| = 5$
$|p| = 5$
$|q| = \sqrt{37}$
$|u| = 4$

d) $\arg z = 27°$
$\arg w = -53°$
$\arg p = -90°$
$\arg q = -171°$
$\arg u = 0°$

11. a) $\bar{z} = a - bi$
b) $z + \bar{z} = 2a \in \mathbb{R}$
c) $z\bar{z} = a^2 + b^2 \in \mathbb{R}$

12. a) $180°$
b) rotate by $180°$; yes

13. a) b)

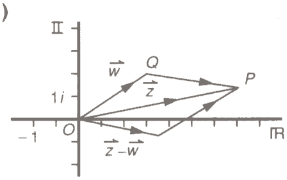


One side of a triangle is shorter than the sum of the other two sides.

c) One side of a triangle is greater than the difference between the other two sides.

14. a) midpoint between z and w
b) divides segment from z to w, in ratio $n : m$

15. b)

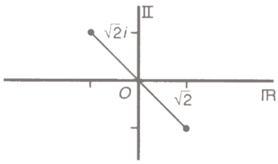


Review Exercises, pages 467–471, continued

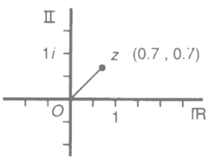
16. **a)** $z^2 - (5 - 2i)z + (-1 - 5i) = 0$
b) $z^2 - [a + c + (b - d)i] + [ac + bd + i(bc - ad)] = 0$

17. **b)** $0, 2, 4 \ldots$ or n

18. $-\sqrt{2} + i\sqrt{2}$ or $\sqrt{2} - i\sqrt{2}$



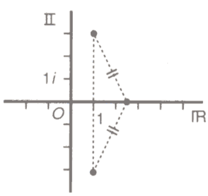
19. **a)** $z^2 = 1$
b)



c) It is true that
$$\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^2 = i,$$
but there is another 'root of i'.

20. $x^2 + y^2 = (x + iy)(x - iy)$ (only in $\mathbb{C}$)

21. **a)** $1 \pm 3i, \frac{5}{2}$
b)



22. **a)** $w^3 = 1$
b) $w^2 = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$
$= \cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3}$
$(w^2)^3 = 1$

24. Yes; e.g. $z = 1$ or $z = i$ are roots of $z^2 - (1 + i)z + i = 0$

25. **a)** i
b) i
c) $\cos 45° + i\sin 45°$ or $\cos 135° - i\sin 135°$
d) -1

26. **b)** $z = 2(\cos\theta + i\sin\theta)$ or $z = \cos\theta - i\sin\theta$

28. **a)** $|z| = y$, $\arg z = 90°$
b) $|z| = x$, $\arg z = 180°$

29. **a)** $\bar{z} = 3(\cos 67° - i\sin 67°)$
b) $\bar{w} = 2(\cos 123° + i\sin 123°)$
c) $zw = 6(\cos 56° - i\sin 56°)$
d) $\bar{z}\,\bar{w} = 6(\cos 56° + i\sin 56°)$
e) $\frac{3}{2}(\cos 190° + i\sin 190°)$ or $\frac{3}{2}(\cos 170° - i\sin 170°)$
f) $\frac{2}{3}(\cos 190° - i\sin 190°)$ or $\frac{2}{3}(\cos 170° + i\sin 170°)$

30. **a)** $z = \sqrt{101}(\cos 6° + i\sin 6°)$
b) $w = \sqrt{65}(\cos 60° - i\sin 60°)$
c) $zw = \sqrt{6565}(\cos 54° - i\sin 54°)$
d) $\frac{z}{w} = \sqrt{\frac{101}{65}}(\cos 66° + i\sin 66°)$
e) $\frac{w}{z} = \sqrt{\frac{65}{101}}(\cos 66° - i\sin 66°)$

31. **a)** $|z| = 2$, $\arg z = 120°$
$|w| = \sqrt{2}$, $\arg w = -135°$
b) $z^3 = 8$
$w^4 = -4$

32. **a)** $2\sqrt{2}(\cos 15° - i\sin 15°)$
b) $\sqrt{2}(\cos 255° + i\sin 255°)$ or $\sqrt{2}(\cos 105° - i\sin 105°)$
c) $\frac{1}{\sqrt{2}}(\cos 105° + i\sin 105°)$

33. **a)** 3θ
b) $\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$
$\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$

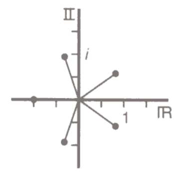
34. **a)** 1
b) $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$
c) $32i$
d) $-\frac{1}{8} - \frac{\sqrt{3}}{8}i$
e) $-\frac{\sqrt{3}}{2} + \frac{i}{2}$
f) i

35. $\cos\theta + i\sin\theta$

36. modulus 1
argument 2θ

38. **b)** $\cos\frac{2k\pi}{6} + i\sin\frac{2k\pi}{6}$, $k \in \{0,1,2,3,4,5\}$
That is,
$1, \frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} + \frac{\sqrt{3}}{2}i,$
$-1, -\frac{1}{2} - \frac{\sqrt{3}}{2}i, \frac{1}{2} - \frac{\sqrt{3}}{2}i$

39. $\cos\left(\pi + \frac{2k\pi}{5}\right) + i\sin\left(\pi + \frac{2k\pi}{5}\right)$, $k \in \{0,1,2,3,4\}$
that is,
$-1, -0.31 \pm 0.95i,$
$0.81 \pm 0.59i$



40. **a)** $|z - 3 - 4i| = 5$
b) If $z = 0$,
$L.S. = |-3 - 4i| = 5 = R.S.$

41. **a)** $(x - 1)^2 + (y + 3)^2 = 1$
b) $x^2 + \left(y - \frac{9}{7}\right)^2 = \frac{25}{49}$

42. circle: centre $\frac{1}{3} - \frac{2}{3}i$
radius $\frac{\sqrt{20}}{3} \doteq 1.49$

43. **a)** $4e^{-\frac{2\pi i}{3}}$
b) $5\sqrt{2}\,e^{-\frac{\pi i}{4}}$

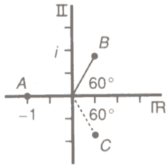
45. **i)** $z = 1 + i, 1 - i, -\frac{1}{2}$
ii) centre $-4 + 3i$, radius 5
$3x - 4y + 24 = 0$

46. **i)** $z = \sqrt{2} + i\sqrt{3}, -2 - i\sqrt{3}, \sqrt{2} - i\sqrt{3}, -\sqrt{2} + i\sqrt{3}$
ii) $\theta = \pm 48.6°$ or $\theta = \pm 131.4°$

48. **a)** $z = \frac{1}{2} \pm i\frac{\sqrt{3}}{2}$
b) $\cos\frac{\pi}{3} \pm i\sin\frac{\pi}{3}$

d) All three roots have modulus 1.

Arguments are $\pi, \frac{\pi}{3}, -\frac{\pi}{3}$



49. i) $z = 2, -1 + 3i, -1 - 3i$

$\sphericalangle A = 90°, \sphericalangle B = 45°, \sphericalangle C = 45°$

arg of $-1 + 3i$ is 1.89

arg of $-1 - 3i$ is -1.89

ii) $\theta = 15°, 75°, 195°, 255°$

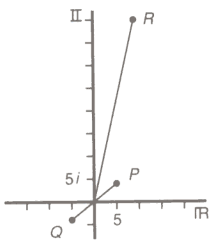
iii) $\theta = 167°$ or $300°$

50. a) $z^2 = (x^2 - y^2) + 2i\,xy$

b) $x = \pm 5, y = \pm 4;$

$z = 5 + 4i$ or $z = -5 - 4i$

c)



d) $OP = OQ = \sqrt{41}$

e) $\sphericalangle POX \doteq 39°$

f) $\sphericalangle POR \doteq 39°$

51. i) $z = 1 + 2i$

$w = 3 - i$

ii) $\frac{z_1}{z_2}$ has modulus $\frac{1}{\sqrt{2}}$,

argument $\frac{5}{12}\pi$

$\frac{1}{z_2^6}$ has modulus $\frac{1}{64}$,

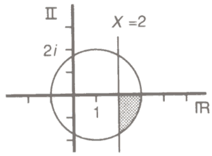
argument π

52. i) $z =$

$$2\left[\cos\left(-\frac{13\pi}{18}\right) + i\sin\left(-\frac{13\pi}{18}\right)\right]$$

$z = 1 + 2i, z = 1 - 2i$

ii)



53. i) $\left(\frac{5}{3}, 0\right)$ radius $\frac{4}{3}$

ii) $\cos 54° = \frac{1}{4}\sqrt{10 - 2\sqrt{5}}$

Problem Supplement,

pages 472–483

4. **a)** no; not coplanar
b) yes; coplanar
c) no; not coplanar

5. **b)** $\vec{a} - 0.5\vec{b}$

6. $\vec{b} + 2\vec{c} + 3\vec{d}$

7. 8.5

8. **a)** $\frac{5}{8}\overrightarrow{OA} + \frac{3}{8}\overrightarrow{OB}$
b) $-\frac{5}{6}\overrightarrow{OA} + \frac{11}{6}\overrightarrow{OB}$

9. **a)** R, K, M collinear; R divides KM externally in ratio 5 : 3; the four points are coplanar
b) (11,4.5,–1)

10. $\overrightarrow{AD} = 3\overrightarrow{AB} + 5\overrightarrow{AC}$

11. 13 : 5

12. **b)** any vector $k\vec{a} + m\vec{b}$, where $21k - 69m = 0$

13. **b)** (3,0,1)

14. $k = -\frac{46}{17}, m = \frac{67}{17}$

16. $2\vec{a} - \vec{b} + \vec{c} - 3\vec{d}$

20. $\overrightarrow{OD} = m(\vec{b} - \vec{c}) + k(\vec{c} - \vec{a})$

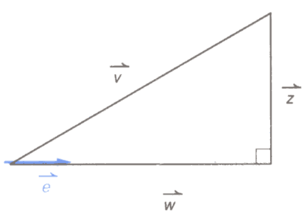
21. $\overrightarrow{BM} = \frac{1}{2}(\vec{a} + \vec{c})$

$\overrightarrow{AM} = \frac{1}{2}(\vec{c} - \vec{a})$

$\overrightarrow{AM}^2 = \overrightarrow{BM}^2$

23. $\overrightarrow{AB} = \vec{b} - \vec{a}$
$\overrightarrow{BC} = \vec{c} - \vec{b}$
$\overrightarrow{OB} \cdot \overrightarrow{AC} = 0$

25.



26. $\overrightarrow{(-2,3)} = \frac{1}{2}\overrightarrow{(1,1)} + \frac{5}{2}\overrightarrow{(-1,1)}$

27. $u_1v_1 + u_2v_2 + u_3v_3 \leqslant \sqrt{(u_1^2 + u_2^2 + u_3^2)(v_1^2 + v_2^2 + v_3^2)}$

29. 5 km/h

30. **a)** $\overrightarrow{(6,6,0)} \cdot \overrightarrow{(1,-1,2)} = 0$
$\Rightarrow$ diagonals perpendicular
b) $\frac{\sqrt{78}}{2} \doteq 4.42$
$\alpha = 32°, \beta = 148°$

33. $|\vec{F}| = 70.7$ N
$\theta = 135°$

34. **a)** 3.91 km/h; bearing 040°
b) 208 m
c) 5 min

35. **a)** bearing 304°
b) 1.66 km/h
c) 9 min

36. **a)** $|\vec{v}| = 375$ km/h
b) bearing 173°

37. **a)** $\vec{r} = \overrightarrow{(2,5,3)} + k\overrightarrow{(2,3,6)}$

$$\begin{cases} x = 2 + 2k \\ y = 5 + 3k \\ z = 3 + 6k \end{cases}$$

$$\frac{x-2}{2} = \frac{y-5}{3} = \frac{z-3}{6}$$

b) $\vec{r} = \overrightarrow{(-2,-1,4)} + t\overrightarrow{(5,3,-2)}$

$$\begin{cases} x = -2 + 5t \\ y = -1 + 3t \\ z = 4 - 2t \end{cases}$$

$$\frac{x+2}{5} = \frac{y+1}{3} = \frac{z-4}{-2}$$

38. **a)** skew
b) $\frac{2}{\sqrt{222}}$

39. $\vec{r} = \overrightarrow{(8,-6,7)} + t\overrightarrow{(8,11,-10)}$

40. $\vec{r} = \overrightarrow{(6,-1)} + k\overrightarrow{(3,5)}$

41. $\left(1 \pm \frac{2}{\sqrt{3}}, 3 \pm \frac{1}{\sqrt{3}}, 2 \mp \frac{2}{\sqrt{3}}\right)$

42. **a)** $\frac{8}{3}$

43. a) c)

44. **a)** 45°
b) 45°; 90°

45. **a)** 3
b) (–1,–6,3)

46. **a)** Line can be on either side of $\overrightarrow{AQ} = \vec{u}$.
b) $\vec{r} = \overrightarrow{(2,1)} + k\overrightarrow{(1,-7)}$
$\vec{r} = \overrightarrow{(2,1)} + t\overrightarrow{(7,1)}$

47. $\vec{r} = \overrightarrow{(-1,2,1)} + k\overrightarrow{(2,0,3)} + s\overrightarrow{(0,1,0)}$

48. $\vec{r} = \overrightarrow{(1,2,3)} + k\overrightarrow{(1,-2,2)} + 5\overrightarrow{(1,2,-3)}$

49. (1,1,3)

50. $11x + 28y + 9z = 10$

51. $14x - 13y + 8z = -18$

52. (1,2,–3)

54. **a)** $k \neq -3, k \neq -1$
b) $k = -3$
c) $k = -1$

55. $x - y + 3z = 3$

56. **a)** $\frac{14}{\sqrt{26}}$
b) $\pm\sqrt{\frac{2511}{46}}$

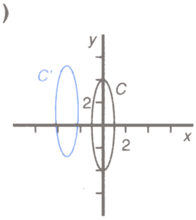
58. One matrix being $m \times n$, the other is $n \times m$.

59. **a)** $(0,1) \rightarrow (0,1)$
$(0,b) \rightarrow (0,b)$
Points on y-axis remain invariant.
b) $(1,0) \rightarrow (1,k)$
$(1,b) \rightarrow (1,b + k)$
Points on $x = 1$ move upward by k.
c) $(a,0) \rightarrow (a,ka)$
Points on x-axis 'dilated' by k, upward.
d) det(s) = 1
same area,
same orientation

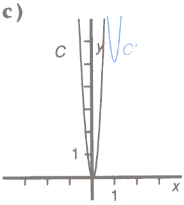
60. **a)** $\vec{i} \rightarrow \begin{bmatrix} \cos 60° \\ \sin 60° \end{bmatrix}$
b) $\vec{j} \rightarrow \begin{bmatrix} \sin 60° \\ -\cos 60° \end{bmatrix}$
c) $R = \begin{bmatrix} \cos 60° & \sin 60° \\ \sin 60° & -\cos 60° \end{bmatrix}$
d) $R = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$

61. $M_{60} = M_{240}$ (same line)

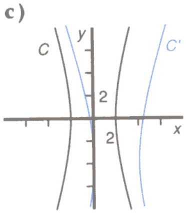
62. **a)** ellipse
b) $16x^2 + y^2 + 96x - 2y + 129 = 0$
c)



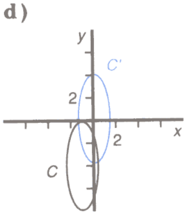
63. a) parabola
b) $16x^2 - 32x - y + 21 = 0$
c)



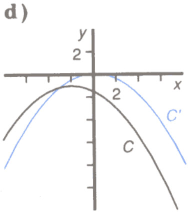
64. a) hyperbola
b) $9x^2 - y^2 - 36x - 8y - 34 = 0$
c)



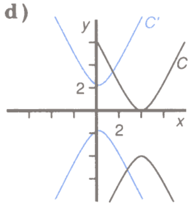
65. a) ellipse
b) $(x,y) \rightarrow (x + 1, y + 4)$
c) $8x^2 + y^2 = 15$
d)



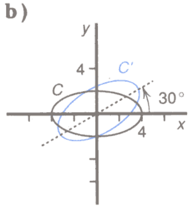
66. a) parabola
b) $(x,y) \rightarrow (x + 2, y + 1)$
c) $x^2 + 8y = 0$
d)



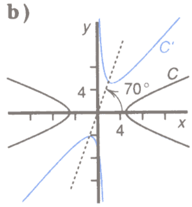
67. a) hyperbola
b) $(x,y) \rightarrow (x - 4, y + 2)$
c) $4x^2 - y^2 = -4$
d)



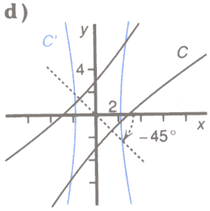
68. a) $1.7x^2 - 2.6xy + 3.3y^2 = 16$
b)



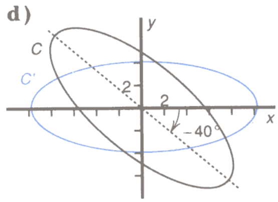
69. a) $-21.6x^2 + 18.6xy - 0.6y^2 = 100$
b)



70. a) hyperbola
b) $45°$
c) $16x^2 - y^2 = 64$
d)



71. a) ellipse
b) $40.0°$
c) $x^2 + 25y^2 = 100$
d)



73. b) 323 160

79. a) $a^5 + 5a^4x + 10a^3x^2 + 10a^2x^3 + 5ax^4 + x^5$
b) $256a^4 - 768a^3 + 864a^2 - 432a + 18$

80. a) $\cos\frac{2k\pi}{9} + i\sin\frac{2k\pi}{9}$, $k \in \{1,2,4,5,7,8\}$
b) $\theta = \frac{2k\pi}{9}$, $k \in \{2,4,5,7,8\}$

82. a) 0 or 3
b) $4w$; or $4w^2$

83. a) $A^{-1} = \frac{1}{9}\begin{bmatrix} 1 & -1 \\ 7 & 2 \end{bmatrix}$
$x = 2, y = 3$
b) $\vec{v} = \begin{bmatrix} 3t_1 - 2t_2 \\ t_1 \end{bmatrix}$
$\lambda = 1$ or 2
c) $\vec{v} \cdot \vec{w} \neq 0$
$t_1 = 1, t_2 = -2$
d) $\vec{z} = \begin{bmatrix} -1 \\ 8 \end{bmatrix}$

84. i) $\vec{b} = \vec{a} + \vec{c}$
iii) $\measuredangle ABC \doteq 48°$
$\measuredangle ABO \doteq 23°$
Adjacent sides have length $\sqrt{14}$ and 3,
$\Rightarrow OABC$ not a rhombus

85. a) midpoint of $[EF] = \frac{1}{4}(\vec{a} + \vec{b} + \vec{c} + \vec{d})$;
midpoints of $[GH]$ and $[LM]$
have same position vector;
the lines are concurrent

86. a) $P_1(3,0,2)$; $P_2(1,-3,-4)$
c) $2x + 3y + 6z = 18$

87. i) a) $\overrightarrow{\left(\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}\right)}$
b) $p = k$;
$|k|$ is the perpendicular distance from O to plane Π_k; Π_k and Π_{-k} are on different sides of the origin
d) $(-1,-1,-3)$
e) $\sqrt{11}$ units

ii) b) $\overrightarrow{(x,y,z)} = \overrightarrow{(3,0,0)} + s\overrightarrow{(0,1,1)}$

88. a) $\lambda = \frac{1}{\sqrt{2}}$

$\mu = -\frac{1}{\sqrt{2}}$

$\vec{u'} = \pm\left(\frac{1}{\sqrt{5}}\vec{j} + \frac{2}{\sqrt{5}}\vec{k}\right)$

b) $t' = 2$

$t = 5$

c) $H\,(1,0,5)$

$K\,(0,2,4)$

89. b) $\vec{OA} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}, \vec{OB} = \begin{bmatrix} 4 \\ -2 \end{bmatrix},$

$\vec{OC} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$

$k = 3, l = 4$

c) $n = -1$

d) $p = -1$

90. a) $x + 4y = 18$

b) $Q(-6,6)$

d) $R'(3b,3b)$, if $R(b,b)$

e) T is a one-way stretch, by a factor of 3, in the direction $x = y$

91. $S = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2} \end{bmatrix}$

92. a) F b) T c) F d) F

93. a) $\vec{w} = \begin{bmatrix} x + 4y \\ 2x - y \end{bmatrix}$

b) $(1 - \lambda)x + 4y = 0$

$2x - (1 + \lambda)y = 0$

For $\lambda = 3$: $\begin{bmatrix} 2y \\ y \end{bmatrix}$ and $\begin{bmatrix} \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{bmatrix}$

For $\lambda = -3$: $\begin{bmatrix} x \\ -x \end{bmatrix}$ and $\begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$

c) $\vec{u} = \begin{bmatrix} x + 4y \\ -2x - y \end{bmatrix}$

No.

94. a) $\vec{u} = \vec{(2,1)}$

$\vec{v} = \vec{(-1,2)}$

b) corresponding to $\vec{u}$: $y = \frac{1}{2}x$

corresponding to $\vec{v}$: $y = -2x$

c) T is the reflection in the line $y = \frac{1}{2}x$.

95. b) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

c) $M^2 = \begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$

$M^4 = \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$

$M^6 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

d) counterclockwise rotations about the orgin, of 120°, 240°, and 0° (identity)

96. $M = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

97. $\lambda_1 = 2, \lambda_2 = 11$

$\vec{e}_1 = \begin{bmatrix} 2 \\ -3 \end{bmatrix}, \vec{e}_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

a) $3x + 2y = 0$ and $y = 3x$

b) $P^{-1}AP = \begin{bmatrix} 2 & 0 \\ 0 & 11 \end{bmatrix}$

98. $M = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$

99. i) $L\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$

$M\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \sin 2\alpha \\ -\cos 2\alpha \end{bmatrix}$

ii) $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$B = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\sin 2\alpha \end{bmatrix}$

iii) $A^2 = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$

i.e. a rotation of 2θ

$B^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

i.e. the identity matrix

100. b) For $a = 0$:

$\vec{(x,y,z)} = k\vec{(1,-1,-1)}$

Two coincident planes meet the plane $x + y = 0$ in a line.

for $a = -1$:

$\vec{(x,y,z)} = \vec{(-1,1,0)} + s\vec{(0,1,1)}$

Three planes intersect in a line.

101. a) $-4d^2 + 26d - 36$

b) $d = 2$ or $d = \frac{9}{2}$

c) For $d = 2$:

$\vec{(-10,4,0)} + s(13,-5,1)$

For $d = 0$: $\left(-\frac{1}{2}, \frac{1}{6}, \frac{5}{6}\right)$

For $d = \frac{9}{2}$: no solution

102. a) -1

b) $\begin{bmatrix} -5 & 4 \\ 4 & -3 \end{bmatrix}$

c) $x = 20, y = -12$

d) 4 unit²

e) $\frac{1}{5}\vec{(4,-3)}$

103. a) $\vec{OC} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}, \vec{OD} = \begin{bmatrix} 2 \\ 8 \end{bmatrix}$

b) $M = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\vec{OE} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}, \vec{OF} = \begin{bmatrix} 8 \\ 2 \end{bmatrix}$

$\det(N) = -4$

$OEF = 4(OAB)$

c) $R = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$S = \begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}$

$S^{-1} = \begin{bmatrix} 0 & -\frac{1}{2} \\ -\frac{1}{2} & 0 \end{bmatrix}$

$\measuredangle GOH \doteq 42°$

APPENDIX

NOTATION

General

$\{\ldots\}$	set notation
$\{x \mid \ldots\}$	set of all x such that
$\varnothing$	empty set
$\in$	belongs to
$\subset$	is a subset of
$\mathbb{N}$	natural numbers
$\mathbb{Z}$	integers
$\mathbb{Q}$	rational numbers
$\mathbb{R}$	real numbers
$\mathbb{C}$	complex numbers
$\mathbb{V}_2$	two-dimensional vector space
$\mathbb{V}_3$	three-dimensional vector space
$\mathbb{V}_{2\times 2}$	vector space of 2×2 matrices
$\Rightarrow$	implies *or* therefore
$\Leftrightarrow$	is equivalent to *or* if and only if
$\parallel$	is parallel to
$\perp$	is perpendicular to

Arithmetic

$\doteq$	equals approximately *
3.204...	(more decimals exist)

Algebra

$C(n,r)$	$(n \text{ choose } r) = \dfrac{n!}{(n-r)!r!}$
$\sum\limits_{r=1}^{n} t_r$	series $t_1 + t_2 + t_3 + \ldots + t_n$
$\lvert x \rvert$	absolute value of x
$f: x \rightarrow y$ or $\{(x,y) \mid y = f(x)\}$	function f maps x onto y

Matrices

a_{ij}	element in the ith row, jth column
I	unit matrix
$0_{2\times 2}$	zero matrix (of dimension 2 by 2)
M^{-1}	inverse of M
$\det(M)$	determinant of M
M^t	transpose of M

*within given restrictions, $\doteq$ may be written $=$

Vectors

$\vec{v}$	vector
$\overrightarrow{AB}$	vector defined by two points
$\lvert\vec{v}\rvert, \lvert\overrightarrow{AB}\rvert$	length or magnitude of a vector
$\overrightarrow{(x,y)}$ or $\begin{bmatrix} x \\ y \end{bmatrix}$	vector as ordered pair
$\overrightarrow{(x,y,z)}$ or $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$	vector as ordered triple
$\vec{i}, \vec{j}, \vec{k}$	standard basis vectors
$\vec{e}_v$	unit vector in the direction of $\vec{v}$
$\vec{u} \cdot \vec{v}$	dot product
$\vec{u} \times \vec{v}$	cross product
$\vec{v}_{PQ}$	velocity of P relative to Q

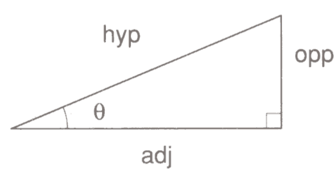
Complex Numbers

$\mathrm{Re}(z)$	real part of z
$\mathrm{Im}(z)$	imaginary part of z
$\bar{z}$	conjugate of z
$\lvert z\rvert = r$	modulus or absolute value of z
$\arg z$	argument of z

The notation used in the International Baccalaureate questions is not always the same as the notation given above. The context of any 'different' notation should make the meaning obvious.

FORMULAS IN TRIGONOMETRY

$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp} \qquad \tan\theta = \frac{opp}{adj}$$



$\cos(90° - \theta) = \sin\theta$
$\sin(90° - \theta) = \cos\theta.$

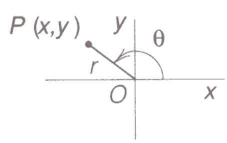
Angles in All Quadrants

The position vector $\overrightarrow{OP} = \overrightarrow{(x,y)}$, where $\lvert\overrightarrow{OP}\rvert = r$, defines an angle θ with the positive x-axis such that

$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}$$



The diagram on the right indicates where the ratios are positive.

Special Angles

$\sin 45^\circ = \dfrac{1}{\sqrt{2}} = \cos 45^\circ$

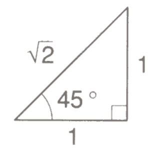
$\tan 45^\circ = 1$

$\sin 30^\circ = \dfrac{1}{2} = \cos 60^\circ$

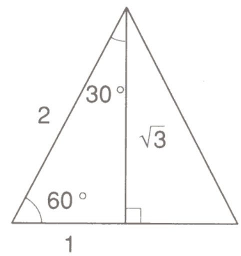
$\sin 60^\circ = \dfrac{\sqrt{3}}{2} = \cos 30^\circ$

$\tan 30^\circ = \dfrac{1}{\sqrt{3}}$

$\tan 60^\circ = \sqrt{3}$



isosceles right triangle



semi-equilateral triangle

Pythagorean Formulas

$\sin^2 A + \cos^2 A = 1$

$\sec^2 A = 1 + \tan^2 A$

$\csc^2 A = 1 + \cot^2 A$

Compound Angles

$\sin(A + B) = \sin A \cos B + \sin B \cos A$

$\sin(A - B) = \sin A \cos B - \sin B \cos A$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2 \sin^2 A$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

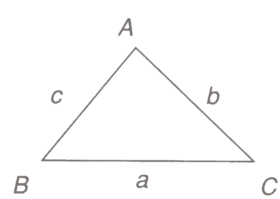
Solution of Triangles

The cosine law

$a^2 = b^2 + c^2 - 2bc \cos A$

The sine law

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$



θ		$\sin\theta$	$\cos\theta$	$\tan\theta$
degree	radian/approx.value			
30	$\frac{\pi}{6} \doteq 0.52$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45	$\frac{\pi}{4} \doteq 0.79$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60	$\frac{\pi}{3} \doteq 1.0$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90	$\frac{\pi}{2} \doteq 1.6$	1	0	undefined
120	$\frac{2\pi}{3} \doteq 2.1$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$
135	$\frac{3\pi}{4} \doteq 2.4$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	-1
150	$\frac{5\pi}{6} \doteq 2.6$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$
180	$\pi \doteq 3.1$	0	-1	0
210	$\frac{7\pi}{6} \doteq 3.7$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
225	$\frac{5\pi}{4} \doteq 3.9$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	1
240	$\frac{4\pi}{3} \doteq 4.2$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$
270	$\frac{3\pi}{2} \doteq 4.7$	-1	0	undefined
300	$\frac{5\pi}{3} \doteq 5.2$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$
315	$\frac{7\pi}{4} \doteq 5.5$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	-1
330	$\frac{11\pi}{6} \doteq 5.8$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$
360	$2\pi \doteq 6.3$	0	1	0

The values of the other ratios can be obtained from the identities $\csc\theta = \frac{1}{\sin\theta}$, $\sec\theta = \frac{1}{\cos\theta}$, and $\cot\theta = \frac{1}{\tan\theta}$

GLOSSARY

This abbreviated glossary provides definitions for mathematical terms as they are used in this text. Consult a mathematics dictionary for more complete or alternative definitions.

Acceleration. The rate at which a speed or velocity is changing.

Angle between a line and a plane. The acute angle between a line and its perpendicular projection in the plane.

Angle between planes. Either the angle between normals to the planes, or the supplement of that angle.

Angle between vectors. The angle between two vectors is the angle between them when they are drawn with a common tail.

Argand diagram. See **Complex plane.**

Argument. If P is the point representing the complex number z in the complex plane, then the argument of z is the angle between OP and the positive real axis.

Associativity. The operation $*$ in the set S has the associative property if, for all $a, b, c \in S$, $a*(b*c) = (a*b)*c$.

Basis. A set of vectors forms a basis for a vector space $\mathbb{V}$ if:
1. it is a linearly independent set and
2. it generates the space; that is, every vector of $\mathbb{V}$ can be expressed as a linear combination of the vectors of the set.

Basis vectors for $\mathbb{V}_2$. Any two linearly independent vectors $\vec{a}$ and $\vec{b}$ form a basis for $\mathbb{V}_2$.

Basis vectors for $\mathbb{V}_3$. Any three linearly independent vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ form a basis for $\mathbb{V}_3$.

Bearing. Direction expressed in degrees measured clockwise from north.

Binary operation. Given a set S, a binary operation $*$ in S combines any two elements of S to give an element of S; that is, if $a, b \in S$, then $a*b \in S$.

Commutativity. The operation $*$ in the set S has the commutative property if, for all $a, b \in S$, $a*b = b*a$.

Complex plane. Complex numbers can be represented geometrically in a complex plane, determined by a real axis (representing all the real numbers) and an imaginary axis. (It is also known as an Argand diagram.)

Complex number. A number that can be expressed as $z = x + iy$, where $x, y \in \mathbb{R}$, and $i^2 = -1$. The real numbers form a subset of complex numbers.

Component. The component of the vector $\vec{u}$ in the direction of $\vec{v}$, where the angle between $\vec{u}$ and $\vec{v}$ is θ, is the scalar $|\vec{u}|\cos\theta$.

Components. If a vector $\vec{v}$ is expressed as $\vec{v} = x\vec{e}_1 + y\vec{e}_2$, where $\vec{e}_1$ and $\vec{e}_2$ are unit vectors, then the scalars x and y are the components of $\vec{v}$ in the directions of $\vec{e}_1$ and $\vec{e}_2$ respectively.

Composition of transformations. The composition of two transformations is the result of applying one transformation after the other.

Conic. A curve such as a circle, parabola, ellipse, or hyperbola that can be obtained from the intersection of a cone and a plane.

Conic, general form.
$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$
For a central conic $g = f = 0$.
If axes are parallel to x-axis and to y-axis, then $h = 0$.

Coordinates. Numbers that locate a point in 2-space or 3-space.

Coplanar vectors. Three vectors are coplanar if directed line segments that represent them can be translated so that all three segments lie in the same plane.

Cross product. The cross product of vectors $\vec{u}$ and $\vec{v}$, making an angle θ, is the vector $|\vec{u}||\vec{v}|\sin\theta\,\vec{e}$, where $\vec{e}$ is a unit vector normal to $\vec{u}$ and $\vec{v}$, such that $\vec{u}$, $\vec{v}$, $\vec{e}$ form a right-handed system.

Degenerate conic. A point, a straight line, or part of a line formed by the intersection of a cone and a plane.

Determinant. The determinant of a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is the real number $ad - bc$.

The value of the 2×2 determinant $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$.

The value of the 3×3 determinant

$$\begin{vmatrix} m & n & p \\ q & r & s \\ t & u & v \end{vmatrix} = m(rv - su) - n(qv - st) + p(qu - rt).$$

Direction. A set of parallel lines with arrows pointing the same way.

Direction cosines of a line. The numbers

$$\cos\alpha = \frac{m_1}{|\vec{m}|},\ \cos\beta = \frac{m_2}{|\vec{m}|},\ \cos\gamma = \frac{m_3}{|\vec{m}|},$$ where α, β, and γ are the angles made with the x, y, and z axes respectively and $\vec{m} = (m_1, m_2, m_3)$ is parallel to the line.

Direction vector of a line. A vector that is collinear with, or parallel to, a line.

Displacement. A directed distance (which can hence be represented by a vector).

Distributivity. In the set S, the operation * is distributive over the operation # if, for all $a, b, c \in S$, $a*(b\#c) = (a*b)\#(a*c)$ and $(b\#c)*a = (b*a)\#(c*a)$.

Dot product. The dot product of vectors $\vec{u}$ and $\vec{v}$, making an angle θ, is the scalar $|\vec{u}||\vec{v}|\cos\theta$.

Dynamics. The study of how objects change their motion under the action of forces.

Equilibrant. Given that $\vec{R}$ is the resultant of a number of force vectors, then the equilibrant of the force vectors is $-\vec{R}$.

Equilibrium. A particle is said to be in equilibrium when the vector sum of all forces acting upon it is zero.

Gravitational force. A particle of mass m kilograms has a gravitational force of mg newtons acting upon it, where g is the acceleration due to gravity. (On earth, $g \doteq 9.8\ m/s^2$.)

Identity. See **Neutral element.**

Image. Given a transformation T and a vector $\vec{v}$, the vector $\vec{v'} = T\vec{v}$ is the image of $\vec{v}$.

Imaginary number. The name describing any number ki where $k \in \mathbb{R}$, and $i^2 = -1$. Imaginary numbers form a subset of complex numbers.

Invariant lines. Lines that are their own images under a particular transformation are known as invariant for that transformation.

Inverse element. $a' \in S$ is the inverse of element $a \in S$ for the operation * if $a'*a = a*a' = e$, where $e \in S$ is the neutral element for *.

Isometry. A transformation that maps a line segment into a congruent line segment.

Linear dependence of two vectors. Two vectors $\vec{a}$ and $\vec{b}$ are linearly dependent if and only if $\vec{a}$ and $\vec{b}$ are collinear.

Linear dependence of three vectors. Three vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ are linearly dependent if and only if m, k, and p exist, *not all equal to* 0, such that $m\vec{a} + k\vec{b} + p\vec{c} = \vec{0}$, $m, k, p \in \mathbb{R}$.

Linear independence of vectors. Vectors that are not linearly dependent are linearly independent.

Linear transformation. A linear transformation T of a vector space is such that, for any vectors $\vec{u}$, $\vec{v}$, and any scalar k:

1. $T(\vec{u} + \vec{v}) = T\vec{u} + T\vec{v}$
2. $T(k\vec{u}) = k(T\vec{u})$

Magnitude. The length or norm of a vector.

Matrix. A rectangular array of numbers. A square matrix can be used as an operator to effect transformations of a vector space.

Modulus. The modulus of the complex number $z = x + iy$ is $\sqrt{x^2 + y^2}$. (It can also be called the length, magnitude, or absolute value of z.)

Natural measure. Angles are commonly measured in degrees. However, the "natural measure" of an angle is defined as the ratio of the arc subtended by the angle and the radius of the circle. (Natural measure is in radians.)

Neutral element. The set S has an neutral element or identity element e with respect to the operation * if for all $x \in S$, $e*x = x*e = x$.

Norm. See **Magnitude.**

Normal vector. A normal vector to a plane (or a line) is a vector that is perpendicular to the plane (or the line).

Orthogonal. Perpendicular.

Orthonormal set. A set of unit vectors that are mutually orthogonal.

Parameter. An arbitrary constant or a variable in a mathematical expression that distinguishes various specific cases. In the parametric equation of a line a parameter determines a point in the line.

Parametric equations of a line.

in 2-space: $x = x_0 + km_1$
$y = y_0 + km_2$

where (x_0, y_0) is a point on the line, $\overrightarrow{(m_1, m_2)}$ is parallel to the line, and k is a parameter.

in 3-space: $x = x_0 + km_1$
$y = y_0 + km_2$
$z = z_0 + km_3$

where (x_0, y_0, z_0) is a point on the line, $\overrightarrow{(m_1, m_2, m_3)}$ is parallel to the line, and k is a parameter.

Parametric equations of a plane.

in 3-space: $x = x_0 + km_1 + su_1$
$y = y_0 + km_2 + su_2$
$z = z_0 + km_3 + su_3$

where (x_0, y_0, z_0) is a point on the plane, $\vec{m} = \overrightarrow{(m_1, m_2, m_3)}$ and $\vec{u} = \overrightarrow{(u_1, u_2, u_3)}$ are vectors parallel to the plane, $\vec{m} \nparallel \vec{u}$, and k and s are parameters.

Particle. An object that is modelled by a single point.

Position vector. If O is the origin and A is any point, then $\overrightarrow{OA}$ is called the position vector of A.

Projection. The projection of the vector $\vec{u}$ in the direction of $\vec{v}$, where the angle between $\vec{u}$ and $\vec{v}$ is θ, is the vector $(|\vec{u}|\cos\theta)\dfrac{\vec{v}}{|\vec{v}|}$

Projections. If a vector $\vec{v}$ is expressed as $\vec{v} = x\vec{e}_1 + y\vec{e}_2$, where $\vec{e}_1$ and $\vec{e}_2$ are unit vectors, then the vectors $x\vec{e}_1$ and $y\vec{e}_2$ are the projections of $\vec{v}$ in the directions of $\vec{e}_1$ and $\vec{e}_2$.

Radian. See **Natural measure.**

Relative velocity. The relative velocity of A from B is the velocity of A as perceived by the observer B.

Resultant. Given two vectors $\vec{a}$ and $\vec{b}$, then the vector $\vec{a} + \vec{b}$ is called the resultant of $\vec{a}$ and $\vec{b}$.

Right-handed system. Three vectors $\vec{u}$, $\vec{v}$, and $\vec{w}$ form a right-handed system if their directions are such that they could be represented respectively by the thumb, the first finger, and the second finger of a right hand. (See diagram, page 11.)

Roots. The roots of an equation in a single variable are the values of the variable that satisfy the equation.

Scalar. A term used to describe real numbers, to distinguish them from vectors.

Singularity. A singular transformation is one that destroys one or more dimensions by its action. Its matrix is also known as singular, and the determinant of this matrix is zero.

Skew lines. Lines in 3-space that are neither parallel nor intersecting.

Statics. The study of forces acting upon objects that are stationary in a given frame of reference.

Supplement. The supplement of an angle of θ^0 is $180^0 - \theta^0$.

Tension. The pulling force in a taut string.

Thrust. The pushing force exerted by a strut.

Transformation. Any action that changes vectors, points, or figures is a transformation of those vectors, points, or figures.

Translation. A transformation in which the image of a figure is obtained by sliding or displacing the original figure without rotation. It can be defined by the vector $\overrightarrow{(h,k)}$ which maps the point (x,y) to the point $(x + h, y + k)$.

Triangle law of vector addition. $\overrightarrow{OS} + \overrightarrow{ST} = \overrightarrow{OT}$

Triangle law of vector subtraction. $\overrightarrow{ST} = \overrightarrow{OT} - \overrightarrow{OS}$

Triangle inequality. Given any three points P, Q, and R, the lengths $PQ + QR \geqslant PR$.

Unit vector. A vector whose length is 1.

Vector. A mathematical entity that can be represented by a directed line segment, or by an ordered n-tuple of numbers, and that obeys a law of addition.

Vector equation of a line. $\vec{r} = \vec{r}_0 + k\vec{m}$, where $\vec{r}$ is the position vector of any point on the line, $\vec{r}_0$ is the position vector of a given point on the line, $\vec{m}$ is a vector parallel to the line, and k is a parameter.

Vector equation of a plane. $\vec{r} = \vec{r}_0 + k\vec{m} + s\vec{u}$, where $\vec{r}$ is the direction vector of any point on the plane, $\vec{r}_0$ is the direction vector of a given point on the plane, $\vec{m}$ and $\vec{u}$ are vectors parallel to the plane, $\vec{m} \nparallel \vec{u}$, and k and s are parameters.

Vector space. A set of vectors, together with the operations of vector addition and multiplication by a scalar.

Work. The product of the component of a force along a distance moved and of the distance moved.

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